MATH 2300-016 QUIZ 9 Name: ____________________________

For these problems, let $f(x) = (1+x)^{1/3}$

1. Find $f'(x)$, $f''(x)$, and $f'''(x)$.

2. What is the maximum M of $|f'''(x)|$ on the interval $[0,1]$?

3. What is $T_2(x)$, the second degree Taylor polynomial for f centered at $x=0$?

4. Use $T_2(x)$ to estimate $\sqrt[3]{2}$.

5. The following theorem describes the difference between the nth Taylor polynomial of f and the function f itself.

Theorem. *Suppose f is $(n+1)$-times differentiable on an open interval containing a and x. Then there exists c between a and x with*

$$f(x) - T_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$$

where T_n is the nth Taylor polynomial of f centered at a.

In particular, taking absolute values and taking an upper bound

$$M \geq \max\{|f^{(n+1)}(t)| \ : \ t \text{ between } a \text{ and } x\},$$

we have the following inequality for the differnence between f and T_n:

$$|f(x) - T_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}.$$

Bound the absolute value of the difference $f(1) - T_2(1) = \sqrt[3]{2} - T_2(1)$ using Taylor's inequality and the bound M on $|f'''(x)|$ you found above.