MATH 2300-016 QUIZ 6 Name: ____________________________

Collaborators (if any):

Due Monday, February 26th at the beginning of class. Submit your work on additional paper, treating this page as a cover sheet. You may use technology and work with with other students. If you work with others, please list their names above. SHOW YOUR WORK!

Determine whether the sequence converges or diverges. If it converges, find its limit.

1. $a_n = \dfrac{e^n + e^{-n}}{e^{2n} - 1}$
2. $b_n = \ln(2n^2 + 1) - \ln(n^2 + 1)$
3. $c_n = \sqrt[n]{2^n + 3^n}$
4. $d_n = \dfrac{\sin(n) \ln n}{n}$
5. $e_n = \left(1 + \dfrac{t}{n}\right)^n$, where t is a constant.

Determine whether the series is convergent or divergent. If it converges find the sum.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$
2. $\displaystyle\sum_{k=1}^{\infty} \ln\left(1 + \frac{1}{k}\right)$
3. $\displaystyle\sum_{m=1}^{\infty} \frac{m(m+2)}{(m+3)^2}$
4. $\displaystyle\sum_{j=1}^{\infty} [(0.8)^{j-1} - (0.3)^j]$

Show the following:

1. For any $\epsilon > 0$, $\displaystyle\lim_{x\to\infty} \frac{\ln x}{x^\epsilon} = 0$. I.e., $\ln x$ grows more slowly than any power of x.
2. For any $p > 0$, $\displaystyle\lim_{x\to\infty} \frac{x^p}{e^x} = 0$. I.e., e^x (or a^x for any $a > 1$) grows more quickly than any power of x.

Try these more challenging problems from the text.

1. [Section 8.2, problem 58]
2. [Section 8.2, problem 68]
   Answer: $11\pi/96$. Hint: If $(r_n)_{n=0}^{\infty}$ is the sequence of radii of the successively smaller circles, then the ratio r_{n+1}/r_n is constant.