MATH 2300-016 QUIZ 5 Name: _____

1. An cylindrical gas tank lies 10 ft underground (its axis parallel to the ground). The tank has a radius of 4 ft and is 12 ft long. What is the work required to empty the tank (pump all the gas to the surface)? The weight of gasoline is 42 lb/ft^3.

   I will use coordinates where the axis of the cylinder goes through $(x, y) = (0, 0)$ and measure depth in terms of y. A cross-section of the tank at a given depth is a rectangle of length 12 ft and width

   $$w(y) = 2\sqrt{16 - y^2} \text{ ft.}$$

   The weight of the gas at a given depth $-4 \leq y \leq 4$ is given by

   $$42 \cdot 12 \cdot w(y) \cdot dy \text{ lbs,}$$

   and is at a depth of $14 - y$ ft below ground. Hence the work required is

   $$W = \int_{-4}^{4} (14 - y) 42 \cdot 12 \cdot 2\sqrt{16 - y^2} dy = 14112 \int_{-4}^{4} \sqrt{16 - y^2} - 504 \int_{-4}^{4} 2y\sqrt{16 - y^2}$$
   $$= 112896\pi \text{ft-lbs.}$$

   [Note that the second integral is zero since the integrand is odd. The first integral can be done by substituting $y = 4\sin\theta$, $dy = 4\cos\theta \; d\theta$.] A quicker way of doing the problem is to recognize that the work required is that of lifting the entire weight of the tank ($12\pi(4)^2 \cdot 42$ lbs) the distance from the center of mass to the surface (14 ft)

   $$W = 14 \cdot 12\pi(4)^2 = 112896\pi \text{ ft-lbs.}$$

2. Find the centroid of the region bounded by the curves

$$y = x,\ y = x^2 - 6.$$

The two curves intersect at $x = -2, 3$ where

$$y = x = x^2 - 6,\ x^2 - x - 6 = 0,\ (x-3)(x+2) = 0.$$

We need to find two moments and area/mass of the region:

$$M_y/\rho = \int_{-2}^{3} x(x - (x^2-6))dx = x^3/3 - x^4/4 + 3x^2\Big|_{-2}^{3} = 125/12,$$
$$M_x/\rho = \int_{-2}^{3} \frac{x + x^2 - 6}{2}(x - (x^2-6))dx = x^3/6 - x^5/10 + 2x^3 - 18x\Big|_{-2}^{3} = -125/3,$$
$$M/\rho = \int_{-2}^{3} (x - (x^2-6))dx = x^2/2 - x^3/3 + 6x\Big|_{-2}^{3} = 125/6.$$

Hence

$$(\bar{x}, \bar{y}) = (M_y/M, M_x/M) = (1/2, -2).$$