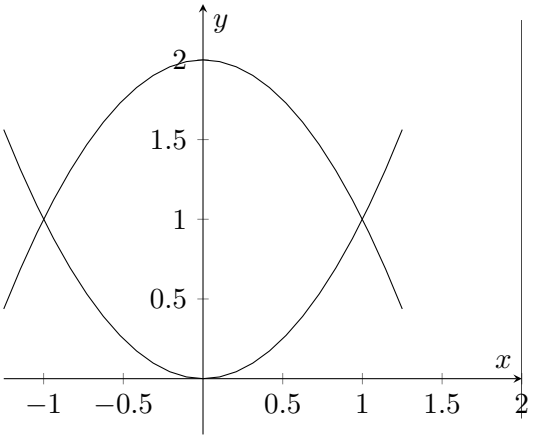
MATH 2300-016 QUIZ 4 Name: _____

Consider the region bounded by the curves

$$y = x^2,\ y = 2 - x^2.$$

Find the volume of the solid formed by revolving the region around the axis $x = 2$.



Using shells is easier, since the height of a cylinder is given by the difference

$$h(x) = (2 - x^2) - x^2.$$

[If we were to use washers, we would need x as a function of y to determine the radii and we'd have to break the integral into two pieces as we integrated with respect to y.]
The radial direction (direction away from the axis of revolution) can be measured by x, so we integrate with respect to x. [Every value $-1 \le x \le 1$ will determine a cylindrical shell.] The radius of a shell determined by x is the distance from x to the axis $x = 2$, hence

$$r(x) = 2 - x$$

is the radius of a typical shell. The volume is then given by

$$V = \int 2\pi r h\ dr = \int_{-1}^{1} 2\pi(2 - x)(2 - x^2 - x^2)dx = 16\pi \int_0^1 (1 - x^2)dx$$
$$= 16\pi(1 - 1^3/3) = \frac{32\pi}{3}.$$

[I used some even/odd symmetry when evaluating the integral if the third equality seems like a jump.]