MATH 2300-015 QUIZ 3 Pt. 1 Name: ____________________________

1. Computer the following integral in two ways (substituting $u = 1 - x^2$ and $x = \sin\theta$):

$$\int \frac{x^3}{\sqrt{1-x^2}}dx$$

With $u = 1 - x^2$, $du = -2xdx$, and $x^2 = 1 - u$, we have

$$\begin{aligned}\int \frac{x^3}{\sqrt{1-x^2}}dx &= -\frac{1}{2}\int \frac{(1-u)du}{\sqrt{u}} = -\frac{1}{2}\int (u^{-1/2} - u^{1/2})du = -u^{1/2} + \frac{u^{3/2}}{3} \\ &= -\frac{(2+x^2)}{3}\sqrt{1-x^2}.\end{aligned}$$

With $x = \sin\theta$, $dx = \cos\theta\ d\theta$, we have

$$\begin{aligned}\int \frac{x^3}{\sqrt{1-x^2}}dx &= \int \frac{\sin^3\theta\cos\theta}{\sqrt{1-\sin^2\theta}}d\theta = \int \sin^3\theta\ d\theta = \int (1-\cos^2\theta)\sin\theta\ d\theta \\ &= \int (u^2 - 1)du = \frac{u^3}{3} - u = \frac{\cos^3\theta}{3} - \cos\theta \\ &= \frac{1}{3}\cos(\arcsin\theta)(\cos^2(\arcsin\theta) - 3) = -\frac{\sqrt{1-x^2}}{3}(x^2+2),\end{aligned}$$

perhaps drawing a triangle to see that $\cos(\arcsin\theta) = \sqrt{1-x^2}$ (for appropriate values of θ).

2. Use integration by parts to reduce $\int x^2 \arcsin x\ dx$ to the integral in problem 1.

With

$$\begin{aligned} u &= \arcsin x, & dv &= x^2dx, \\ du &= \tfrac{dx}{\sqrt{1-x^2}}, & v &= x^3/3, \end{aligned}$$

we get

$$\int x^2 \arcsin x\ dx = \frac{x^3}{3}\arcsin x - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}.$$

The resulting integral is the integral from problem 1, so that

$$\int x^2 \arcsin x\ dx = \frac{x^3}{3}\arcsin x + \frac{(2+x^2)}{9}\sqrt{1-x^2}.$$