MATH 2300-016 QUIZ 1 Name: _____

1. Integrate the following.

(a) $\int (3x+2)^5 dx$

With $u = 3x+2$, $du = 3dx$, we get

$$\int (3x+2)^5 dx = \frac{1}{3}\int u^5 du = \frac{u^6}{18} + C = \frac{(3x+2)^6}{18} + C.$$

(b) $\int_1^{e^\pi} \frac{\cos(\ln y)}{y} dy$

With $u = \ln y$, $du = dy/y$, we get

$$\int_1^{e^\pi} \frac{\cos(\ln y)}{y} dy = \int_{\ln(1)}^{\ln(e^\pi)} \cos u \; du = \sin u \Big|_0^\pi = \sin(\pi) - \sin(0) = 0 - 0 = 0.$$

(c) $\int \frac{e^z}{\sqrt{1+2e^z}} dz$

With $u = 1 + 2e^z$, $du = 2e^z dz$, we get

$$\int \frac{e^z}{\sqrt{1+2e^z}} dz = \frac{1}{2}\int \frac{du}{\sqrt{u}} = \sqrt{u} + C = \sqrt{1+2e^z} + C.$$

2. What are the following derivatives?

   (a) $\dfrac{d}{d\theta}\sec\theta$

   The derivative is $\sec\theta\tan\theta$ (which you can get by differentiating $\sec\theta = 1/\cos\theta$).

   (b) $\dfrac{d}{dx}\arctan x$

   The derivative is $\frac{1}{1+x^2}$ (which you can get by differentiating $\tan(\arctan x) = x$).

3. Integrate the following.

   (a) $\displaystyle\int \frac{dx}{x^2+2x+2}$ (Hint: complete the square in the denominator first.)

   We have

   $$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1}$$

   and with $u = x+1$, $du = dx$, we get

   $$\int \frac{dx}{(x+1)^2+1} = \int \frac{du}{1+u^2} = \arctan u + C = \arctan(x+1) + C.$$

   (b) $\displaystyle\int_0^{\pi/3} \sec^3\theta\tan\theta d\theta$

   We have

   $$\int_0^{\pi/3} \sec^3\theta\tan\theta d\theta = \int_0^{\pi/3} (\sec\theta)^2\sec\theta\tan\theta\ d\theta$$

   so wiith $u = \sec\theta$, $du = \sec\theta\tan\theta d\theta$, we get

   $$\int_0^{\pi/3} (\sec\theta)^2\sec\theta\tan\theta\ d\theta = \int_{\sec(0)}^{\sec(\pi/3)} u^2\ du = \frac{u^3}{3}\Big|_1^2 = 7/3.$$