MATH 2300-016 QUIZ 13 Due Tuesday, April 24th Name: ______________________

1. Recall Newton's law of cooling: the rate of change in temperature of an object is proportional to the difference in temperature between the object and its surroundings,

$$\frac{dT}{dt} = k(T - T_s),$$

where $T(t)$ is temperature as a function of time, k is the proportionality constant, and T_s is the constant surrounding temperature.

Suppose a cup of coffee is 200°F when it is poured and has cooled to 190°F after one minute in a room at 70°F. When will the coffee reach 150°F? What will the temperature of the coffee be after it sits for 30 minutes?

2. The following variation on the logistic equation models logistic growth with constant harvesting:

$$\frac{dP}{dt} = kP(1 - P/M) - c.$$

For this problem consider the specific instance

$$\frac{dP}{dt} = 0.08P(1 - P/1000) - 15,$$

modeling fish population in a pond where 15 fish per week are caught (time t in weeks).

(a) What are the equilibrium solutions to the differential equation in part (i.e. what are the constant solutions)?

(b) Find the general solution of the differential equation. [Integrate using partial fractions. You should get something equivalent to $P(t) = \dfrac{750 - 250Ce^{-t/25}}{1 - Ce^{-t/25}}$ where C is an arbitrary constant.]

(c) Find and interpret the solutions with initial conditions $P(0) = 200, 300$.

Here's a "heads-up" concerning material for the rest of the semester. You will definitely be responsible for the material in sections 1.7, 3.4, and 6.4 concerning parametric curves, slopes/tangents to parametric curves, and arclength of parametric curves. [You may also want to look at sections 9.1. 9.2, and 9.5 concerning 3D coordinates and vectors, and 10.1, 10.2, 10.3 concerning some simple vector calculus - this is mostly optional.] We will also be covering the material in appendices H.1 and H.2, polar coordinates and arclength/area in polar coordinates.