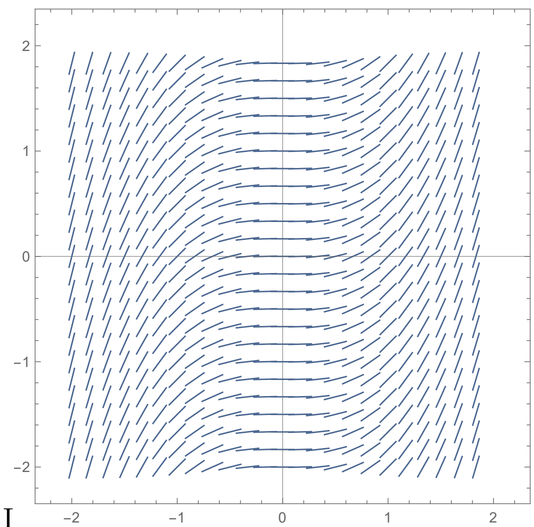
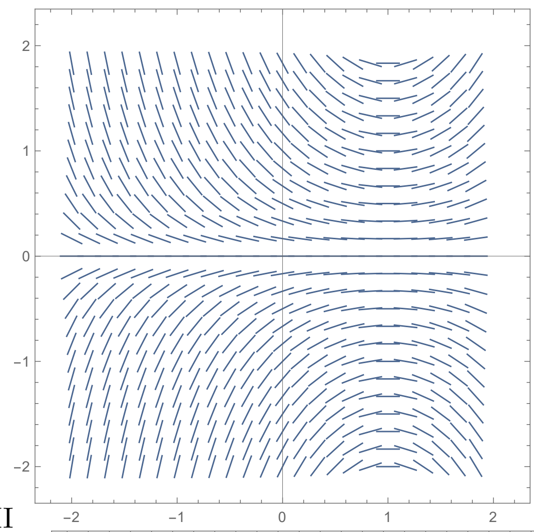
MATH 2300-016 QUIZ 12 Name: ____________________________

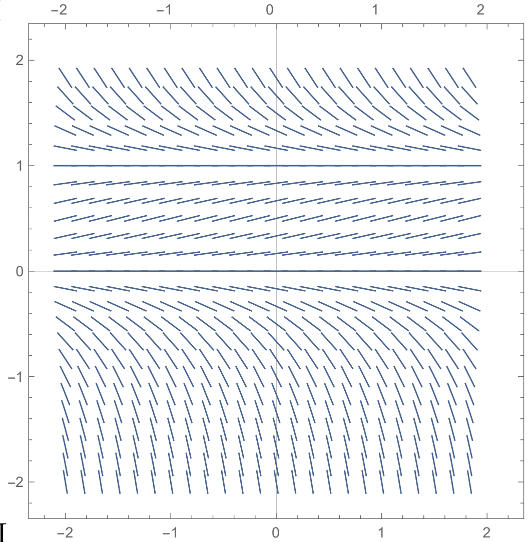
1. Match the slope fields below (labeled I, II, III, IV) with the differential equations below.



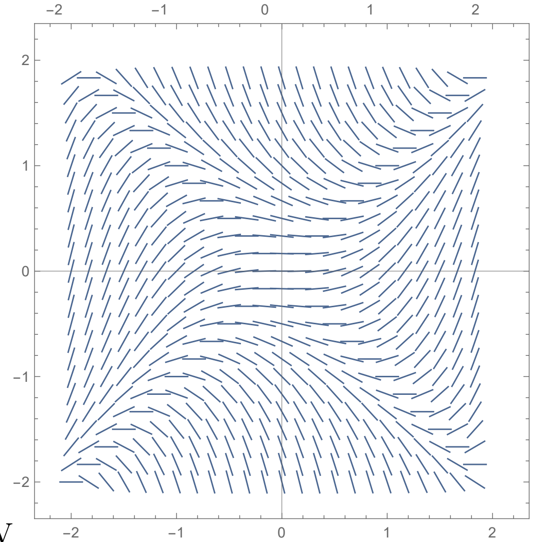
I



II



III



IV

$$\underline{I} \quad \frac{dy}{dx} = x^2$$

$$\underline{III} \quad \frac{dy}{dx} = y - y^2$$

$$\underline{IV} \quad \frac{dy}{dx} = x^2 - y^2$$

$$\underline{II} \quad \frac{dy}{dx} = xy - y$$

2. Use Euler's method with step size 1/2 to approximate $y(2)$ where y is a solution of the initial value problem

$$y' = x - y, \; y(0) = 1,$$

filling in the information in the table below.

n	x_n	y_n	$y'(x_n)$
0	0	1	0-1 =-1
1	1/2	1-1/2 =1/2	1/2-1/2 =0
2	1	1/2+0 =1/2	1-1/2 =1/2
3	3/2	1/2+1/4 =3/4	3/2-3/4 =3/4
4	2	3/4+3/8 =9/8	

Hence $y(2) \approx 9/8$.

3. Solve the following initial value problems.

(a) $y' + y^2 \sin x = 0, \; y(0) = -1/2$

Solution. Rearranging, we have

$$\frac{dy}{dx} = -y^2 \sin x, \; \frac{dy}{y^2} = -\sin x dx$$

so that

$$\begin{aligned} \int \frac{dy}{y^2} &= -\int \sin x dx \\ -\frac{1}{y} &= \cos x + C \\ y &= \frac{-1}{\cos x + C}. \end{aligned}$$

If $y(0) = -1/2$ then $C = 1$ and the solution to the initial value problem is

$$y(x) = \frac{-1}{1 + \cos x}.$$

(b) $y' = \dfrac{x^2}{y(1+x^3)}, \; y(0) = -1$

Solution. Separating variables gives

$$y \; dy = \frac{x^2}{1+x^3} dx.$$

Integrating, we obtain

$$\begin{aligned} \int y \; dy &= \int \frac{x^2}{1+x^3} dx, \\ \frac{y^2}{2} &= \frac{1}{3} \ln |1 + x^3| + C, \\ y &= \pm \sqrt{\frac{2}{3} \ln |1 + x^3| + C}. \end{aligned}$$

If $y(0) = -1$, then we must have $C = 1$ and the negative square root,

$$y(x) = -\sqrt{\frac{2}{3} \ln |1 + x^3| + 1}$$