MATH 2300-016 QUIZ 11 (take home, due Monday 4/9) Name: ____________________

1. Express
$$\int_0^1 \sin(x^2)dx$$
as an infinite series. Use the first two terms of the series to approximate the definite integral and bound the error using the alternating series remainder estimate.

2. Starting with the geometric series, $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$, show that

$$\pi = 2\sqrt{3}\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}.$$

[Hint: Evaluate $\arctan(x)$ at $1/\sqrt{3}$.]

3. Find the Taylor series for $\cos x$ centered at $x = \pi/3$ in two ways: from scratch and using the trigonometric identity

$$\cos(x) = \cos(x - \pi/3 + \pi/3) = \cos(x - \pi/3)\cos(\pi/3) - \sin(x - \pi/3)\sin(\pi/3)$$

along with knowledge of the Taylor series for $\sin x$, $\cos x$ centered at $x = 0$.

4. Show that

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

for all x, i.e. use Taylor's inequality to show that the difference between $\cos x$ and its ($2N$th degree) Taylor polynomial

$$\cos x - \sum_{n=0}^{N} (-1)^n \frac{x^{2n}}{(2n)!}$$

goes to zero as $N \to \infty$.

5. Find $T_4(x)$, the fourth degree Taylor polynomial for $f(x) = \sqrt{x}$ centered at $x = 9$, and use $T_4(x)$ to estimate $\sqrt{10}$. Use Taylor's inequality to bound the error in the approximation.