Divergence Test

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_n a_n$ diverges.

Absolute Convergence

If $\sum_n |a_n|$ converges, then $\sum_n a_n$ converges.

Integral Test

If $(a_n)_{n=1}^{\infty} = (f(n))_{n=1}^{\infty}$ with $f(x)$ positive and decreasing on $[1, \infty)$, then

$$\sum_{n=1}^{\infty} a_n \text{ converges } \Leftrightarrow \int_1^{\infty} f(x)dx \text{ converges.}$$

Comparison Test(s)

Suppose $0 \leq a_n \leq b_n$ are two sequences of positive terms. Then

$$\sum_n a_n \text{ diverges } \Rightarrow \sum_n b_n \text{ diverges,}$$
$$\sum_n b_n \text{ converges } \Rightarrow \sum_n b_n \text{ converges.}$$

If $0 \leq a_n$, b_n are two sequences of positive terms and $\lim_{n\to\infty} \frac{a_n}{b_n} = 1$ then

$$\sum_n a_n \text{ converges } \Leftrightarrow \sum_n b_n \text{ converges.}$$

Ratio Test (Root Test)

If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ then

$$\sum_n a_n \begin{cases} \text{converges} & 0 \leq L < 1 \\ \text{diverges} & L > 1 \\ ??? & L = 1 \end{cases}.$$

If $\lim_{n\to\infty} |a_n|^{1/n} = L$ then

$$\sum_n a_n \begin{cases} \text{converges} & 0 \leq L < 1 \\ \text{diverges} & L > 1 \\ ??? & L = 1 \end{cases}.$$

Alternating Series Test

Suppose $0 \leq b_n$ is a sequence of positive terms decreasing to zero:

$$b_n \geq b_{n+1}, \ \lim_{n\to\infty} b_n = 0.$$

Then $\sum_n (-1)^n b_n$ converges.