MATH 2300-015 QUIZ 9 Due Tuesday, October 31st Name: ______________________

1. Starting with the geometric series, $\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $|x| < 1$, show that

$$\pi = 2\sqrt{3}\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}.$$

[Hint: Evaluate $\arctan(x)$ at $1/\sqrt{3}$.]

2. What is the radius of convergence of $\sum_{n=0}^{\infty} \dfrac{n^n}{n!} x^n$?

3. This problem will use power series to solve the initial value problem

$$y' = y,\ \ y(0) = 1.$$

   (a) Suppose $y(x) = \sum_{n=0}^{\infty} c_n x^n$ is a solution. Differentiate y term-by-term and equate the coefficients of x^n on both sides of $y' = y$ to determine c_n.

   (b) Show that the resulting series converges for all values of x and differentiate term-by-term to verify that $y' = y$.