MATH 2300-015 QUIZ 8 Due Monday, October 23rd Name: ______________________

Below are terse solutions for the problems. Your solutions should be MUCH more detailed.

1. Determine whether the following series converge or diverge. If a series converges and the terms are not eventually positive, determine whether or not the convergence is absolute

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

   Converges by direct comparison to $\sum_n \frac{1}{n^2}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

   Diverges by limit comparison to $\sum_n \frac{1}{n}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5^n}$

   Converges by the ratio test ($|a_{n+1}/a_n| \to 1/5$).

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

   Converges conditionally by the alternating series test.

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

   Diverges by the integral test.

   (f) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

   Diverges by the divergence test ($a_n \to \ln(1/3)$).

   (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{\sqrt{n}}{n+1}$

   Converges conditionally by the alternating series test.

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n)}{1+(1.2)^n}$

   $\sum_n |a_n|$ converges by direct comparison to $\sum_n (1/1.2)^n$, a convergent geometric series.

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}{5^n n!}$

   Conveges by the ratio test ($|a_{n+1}/a_n| \to 2/5$).

   (j) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+n}{3n}\right)^n$

   Converges by limit comparison to $\sum_n (1/3)^n$ or by the ratio test ($|a_{n+1}/a_n| \to 1/3$).

   (k) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(2n+1)!}$

   Converges by the ratio test ($|a_{n+1}/a_n| \to 0$).

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{8^n}{n!}$

Converges by the ratio test ($|a_{n+1}/a_n| \to 0$).

(m) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2^n}{n^2}$

Diverges by the divergence test.

(n) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

Converges conditionally by the alternating series test ($\cos(n\pi) = (-1)^n$).

(o) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan(1/n)}{n^{3/2}}$

Converges by direct comparison to $(\pi/4)\sum_n n^{-3/2}$ or by limit comparison to $\sum_n 1/n^{5/2}$ ($\tan(x) \approx x$ for x small).

(p) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2+\sin n}$

Diverges by the divergence test.

(q) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n^2)$

Converges by limit comparison to $\sum_n 1/n^2$.

(r) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos(1/n^2)$

Diverges by the divergence test.

(s) $\displaystyle\sum_{n=1}^{\infty} 2^{-\ln n}$

Diverges. This is a p-series in disguise, $2^{-\ln n} = 1/n^{\ln 2}$, $p = \ln 2 < 1$.

(t) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

Converges by the integral test.

2. (a) The sum is $-\ln 2$ (the Nth partial sum is $-\ln 2 + \ln(1 + 1/N)$).
   (b) The sum is $11/45$ (the sum of two geometric series).

3. (a) $N \geq 22027$ suffices.
   (b) $N \geq e^{e^{10}} - 1 \approx 9.38 \times 10^{9565}$