MATH 2300-015 QUIZ 8 Due Monday, October 23rd Name: ____________________

[Don't forget toreview sectoin 6.6 which is included on the exam (work, hydrostatic pressure, center of mass). You should also look at your homework for October 26th as this covers material on the exam.]

DIRECTIONS FOR CREDIT: Carefully write out solutions for five items from problem 1 along with solutions to problems 2 and 3.

1. Determine whether the following series converge or diverge. If a series converges and the terms are not eventually positive, determine whether or not the convergence is absolute. Carefully explain your reasoning and be sure to verify the hypotheses of any tests you may use.

   You may want to ask yourself the following (and in this order!):

   - What is the rate of growth of the terms, $|a_n|$? What do the terms $|a_n|$ look like for large n? Do the terms go to zero?
   - Does the series remind you of a known convergent or divergent series, e.g. p-series, geometric series? Can you use the direct or limit comparison on $\sum_n |a_n|$?
   - Do the terms agree with a function you can integrate, $|a_n| = f(n)$? Try the integral test.
   - Are the terms of the series defined multiplicatively or does the series look geometric? Try the ratio test.
   - If the terms of the series are alternating and none of the above apply to $\sum_n |a_n|$, can you apply the alternating series test?

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{5^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right)$

   (g) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{\sqrt{n}}{n+1}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n)}{1+(1.2)^n}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{5^n n!}$

(j) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+n}{3n}\right)^n$

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(2n+1)!}$

(l) $\displaystyle\sum_{n=1}^{\infty} \frac{8^n}{n!}$

(m) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}2^n}{n^2}$

(n) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

(o) $\displaystyle\sum_{n=1}^{\infty} \frac{\tan(1/n)}{n^{3/2}}$

(p) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2+\sin n}$

(q) $\displaystyle\sum_{n=1}^{\infty} \sin(1/n^2)$

(r) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos(1/n^2)$

(s) $\displaystyle\sum_{n=1}^{\infty} 2^{-\ln n}$

(t) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

2. Find the values of the following series telescoping or geometric series.

   (a) $\displaystyle\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)$

   (b) $\displaystyle\sum_{n=4}^{\infty} \frac{5 \cdot 2^{n-3} + 2 \cdot 3^{n-5}}{3 \cdot 5^{n-2}}$

3. Use the integral or alternating series test remainder estimate for the following problems, first showing that the series in question actually converges.

   (a) How many terms N of the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ can we use to guarantee the remainder

   $$R_N = \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} - \sum_{n=2}^{N} \frac{1}{n(\ln n)^2}$$

   is less than 0.1?

   (b) How many terms N of the series $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(\ln n)}$ can we use to guarantee the remainder

   $$R_N = \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(\ln n)} - \sum_{n=2}^{N} \frac{(-1)^n}{\ln(\ln n)}$$

   is less than 0.1?