MATH 2300-015 QUIZ 3 Name: ____________________________

Due Tuesday, September 19th at the beginning of class. Please use additional paper as necessary to submit CLEAR and COMPLETE solutions.

1. For which values of p does $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^p}$ converge/diverge? Find the value of the improper integral when it is convergent.

2. For what values of p does the improper integral $\displaystyle\int_0^1 \frac{dx}{x^p}$ converge?

3. First, show that $\displaystyle\int_0^\infty \frac{dx}{x^3+1}$ converges by comparison. Second, find the value of the improper integral. (You should get $\frac{2\pi}{3\sqrt{3}}$).

4. Find the value of C for which the following improper integral converges and evaluate the integral for this value of C:
$$\int_0^\infty \left(\frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2}\right) dx.$$
[Note that the integral of each summand separately is divergent, but the right choice of C gives "cancellation" and a convergent integral.]

5. Recall Simpson's rule for approximating a definite integral:
$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1)} + f(x_n)),$$
where
$$\Delta x = \frac{b-a}{n},\ x_i = a + i\Delta x,\ n \text{ even}.$$
(This estimate is obtained by approximating f piecewise on $[x_{2k}, x_{2k+2}]$ by the unique quadratic through the three points $(x_{2k}, f(x_{2k}))$, $(x_{2k+1}, f(x_{2k+1}))$, and $(x_{2k+2}, f(x_{2k+2}))$. Cf. pp. 406-410 of the text.)
A bound for the error
$$E_S := \int_a^b f(x)dx - S_n$$
is given by
$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$
where K is any bound for the fourth derivative of f on the interval $[a, b]$,
$$K \geq |f^{(4)}(x)|,\ x \in [a, b].$$
Using the above, find a value of n large enough to guarantee $|E_S| < 10^{-6}$ when approximating the integral
$$\int_0^1 e^{-x^2} dx.$$
[Note: A computer gives $f^{(4)}(x) = 4e^{-x^2}(4x^4 - 12x^2 + 3)$ when $f(x) = e^{-x^2}$. Use the methods of Calc 1 to find the max/min of the fourth derivative on the interval $[0, 1]$, and use this to determine a value for K.]