MATH 2300-015 QUIZ 1 Name: ____________________________

1. Integrate the following with a substitution, $\int f(u(x))u'(x)dx = \int f(u)du$.

   (a) $\displaystyle\int \frac{dx}{x^2+2x+2}$ (Hint: complete the square in the denominator first.)

   We have $x^2+2x+2 = (x+1)^2+1$. Making the substitution $u = x+1$, $du = dx$ gives

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{(x+1)^2+1} = \int \frac{du}{1+u^2}$$
$$= \arctan u + C = \arctan(x+1) + C.$$

   As an aside, any quadratic ax^2+bx+c can be written as $a(x+h)^2+k$, where

$$h = \frac{b}{2a},\ k = c - \frac{b^2}{4a}.$$

   (b) $\displaystyle\int_0^{\pi/3} \sec^3\theta \tan\theta d\theta$

   With $u = \sec\theta$, $du = \sec\theta\tan\theta d\theta$, $u(0) = \sec(0) = 1/\cos(0) = 1$, $u(\pi/3) = 1/\cos(\pi/3) = 2$, we have

$$\int_0^{\pi/3} \sec^3\theta\tan\theta d\theta = \int_0^{\pi/3} \sec^2\theta\sec\theta\tan\theta d\theta = \int_1^2 u^2 du$$
$$= u^3/3\Big|_1^2 = 7/3.$$

   You could also write the integrand in terms of $\sin\theta$ and $\cos\theta$,

$$\int_0^{\pi/3} \sec^3\theta\tan\theta d\theta = \int_0^{\pi/3} \frac{\sin\theta}{\cos^4\theta} d\theta,$$

   and substitute $u = \cos\theta$, $du = -\sin\theta d\theta$ to get

$$\int_0^{\pi/3} \frac{\sin\theta}{\cos^4\theta} d\theta = -\int_1^{1/2} u^{-4} du = \frac{u^{-3}}{3}\Big|_1^{1/2} = 7/3.$$

2. Integrate the following by parts, $\int u dv = uv - \int v du$.

   (a) $\displaystyle\int \frac{3x}{e^{2x}} dx$

   One could directly integrate by parts, or first make a substitution. With $y = -2x$ (and $x = -y/2$), $dy = -2dx$, we have

$$\int \frac{3x}{e^{2x}} dx = \frac{3}{4} \int y e^y dy.$$

   Integrating by parts with

$$u = y,\ du = dy,\ dv = e^y dy,\ v = e^y,$$

   we get

$$\int \frac{3x}{e^{2x}} dx = \frac{3}{4} \int y e^y dy = \frac{3}{4} \left(y e^y - \int e^y dy \right)$$
$$= -\frac{3}{4}(2x - 1)e^{-2x} + C.$$

   (b) $\displaystyle\int_1^e \ln x dx$

   With

$$u = \ln x,\ du = dx/x,\ dv = dx,\ v = x$$

   we get

$$\int_1^e \ln x dx = x \ln x\Big|_1^e - \int_1^e dx = (x \ln x - x)\Big|_1^e = 1.$$

3. Integrate $\int \arcsin z dz$ (Hint: integrate by parts using $u = \arcsin z$, $dv = dz$, then use a substitution on the resulting integral.)

   With

$$u = \arcsin z,\ du = \frac{dz}{\sqrt{1 - z^2}},\ dv = dz,\ v = z$$

   we get

$$\int \arcsin z dz = z \arcsin z - \int \frac{z dz}{\sqrt{1 - z^2}}.$$

   Making the substitution $w = 1 - z^2$, $dw = -2z$, the above integral becomes

$$\int \frac{z dz}{\sqrt{1 - z^2}} = -\frac{1}{2} \int \frac{dw}{\sqrt{w}} = -\sqrt{w} = -\sqrt{1 - z^2}.$$

   Hence

$$\int \arcsin z dz = z \arcsin z + \sqrt{1 - z^2} + C.$$