MATH 2300-015 QUIZ 13 (in class) Name: ____________________________

1. For $(x, y) = (1, -\sqrt{3})$ in Cartesian coordinates, find two representations (r, θ) in polar coordinates with $-\pi \leq \theta \leq \pi$.

   [First, draw a picture of where $(1, -\sqrt{3})$ is.] We have

$$r^2 = (1)^2 + (-\sqrt{3})^2 = 4, \ \tan(\theta) = -\sqrt{3} = \frac{-\sqrt{3}/2}{1/2} = \frac{\sqrt{3}/2}{-1/2}.$$

   Hence $\theta = 2\pi/3$ or $-\pi/3$. The corresponding r-values are -2 and 2 respectively. We get

$$(r, \theta) = (-2, 2\pi/3), \ (2, -\pi/3).$$

   [Does this match the picture you drew?]

2. Find the Cartesian coordinates (x, y) of the point with polar coordinates $(r, \theta) = (-\sqrt{2}, 3\pi/4)$.

   [Draw a picture of where this point is.] We have

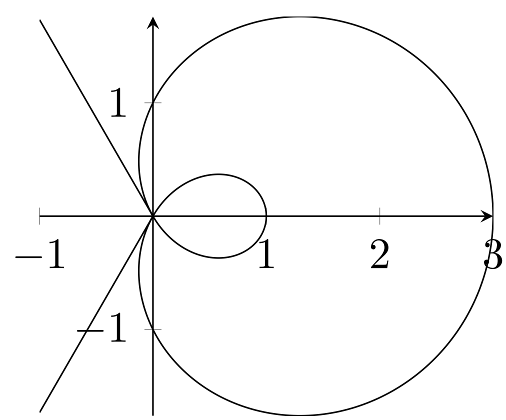
$$x = r\cos\theta = -\sqrt{2}(-1/\sqrt{2}) = 1, \ y = r\sin\theta = -\sqrt{2}(1/\sqrt{2}) = -1.$$

   [Does this match your picture?]

3. Sketch the graph of the polar curve $r = 1 + 2\cos\theta$, $0 \leq \theta \leq 2\pi$. Indicate the angles at which $r = 0$.

   We have

$$r = 0 \Rightarrow \cos\theta = -1/2 \Rightarrow \theta = 2\pi/3, 4\pi/3.$$



4. Consider the parametric curve

$$(x(t), y(t)) = (e^t \cos t, e^t \sin t),\ 0 \le t \le \pi.$$

(a) For what value(s) of t does the curve have a horizontal tangent line?
The slope of the tangent line is

$$dy/dx = \frac{dy/dt}{dx/dt}$$

which is zero when $dy/dt = 0$. We have

$$dy/dt = e^t \cos t + e^t \sin t = e^t(\cos t + \sin t)$$

so that $dy/dt = 0$ when $\tan t = -1$, i.e. $t = 3\pi/4$ (the only possible value in $[0, \pi]$).

(b) Find the length of the curve.
The arclength is

$$L = \int_0^\pi \sqrt{(dx/dt)^2 + (dy/dt)^2} dt = \int_0^\pi \sqrt{(e^t(\cos t - \sin t))^2 + (e^t(\cos t + \sin t))^2} dt$$
$$= \int_0^\pi e^t \sqrt{2} dt = \sqrt{2}(e^\pi - 1).$$