1. Solve the following initial value problems.

   (a) $y' + y^2 \sin x = 0$, $y(0) = -1/2$

   (b) $y' = \dfrac{x^2}{y(1+x^3)}$, $y(0) = -1$

2. Suppose $y(x)$ is the solution to the initial value problem

$$y' = x^2 - y^2, \ y(0) = 1.$$

   Use Euler's method (step size 0.1) to approximate $y(0.5)$.

3. Use the third degree Taylor polynomial (centered at zero) for $f(x) = \ln(1+x)$ to estimate $\ln(2)$ and use Taylor's inequality to give bounds on the error.

4. Recall Newton's law of cooling: the rate of change in temperature of an object is proportional to the difference in temperature between the object and its surroundings,

$$\frac{dT}{dt} = k(T - T_s),$$

   where $T(t)$ is temperature as a function of time, k is the proportionality constant, and T_s is the constant surrounding temperature.

   Suppose a cup of coffee is 200°F when it is poured and has cooled to 190°F after one minute in a room at 70°F. When will the coffee reach 150°F? What will the temperature of the coffee be after it sits for 30 minutes?

5. The following variation on the logistic equation models logistic growth with constant harvesting:

$$\frac{dP}{dt} = kP(1 - P/M) - c.$$

   For this problem consider the specific instance

$$\frac{dP}{dt} = 0.08P(1 - P/1000) - 15,$$

   modeling fish population in a pond where 15 fish per week are caught (time t in weeks).

   (a) What are the equilibrium solutions to the differential equation in part (i.e. what are the constant solutions)?

   (b) Find the general solution of the differential equation. [Integrate using partial fractions. You should get $P(t) = \dfrac{750 - 250Ce^{-t/25}}{1 - Ce^{-t/25}}$ where C is an arbitrary constant.]

   (c) Find and interpret the solutions with initial conditions $P(0) = 200, 300$.

Here's a "heads-up" concerning material for the rest of the semester. You will definitely be responsible for the material in sections 1.7, 3.4, and 6.4 concerning parametric curves, slopes/tangents to parametric curves, and arclength of parametric curves. [You may also want to look at sections 9.1. 9.2, and 9.5 concerning 3D coordinates and vectors, and 10.1, 10.2, 10.3 concerning some simple vector calculus - this is mostly optional.] We will also be covering the material in appendices H.1 and H.2, polar coordinates and arclength/area in polar coordinates.

The third midterm (Monday, December 4th) will cover:

- 8.5, 8.6, 8.7 - power series and Taylor series,

- 7.1, 7.2, 7.3, 7.4, 7.5 - differential equations.