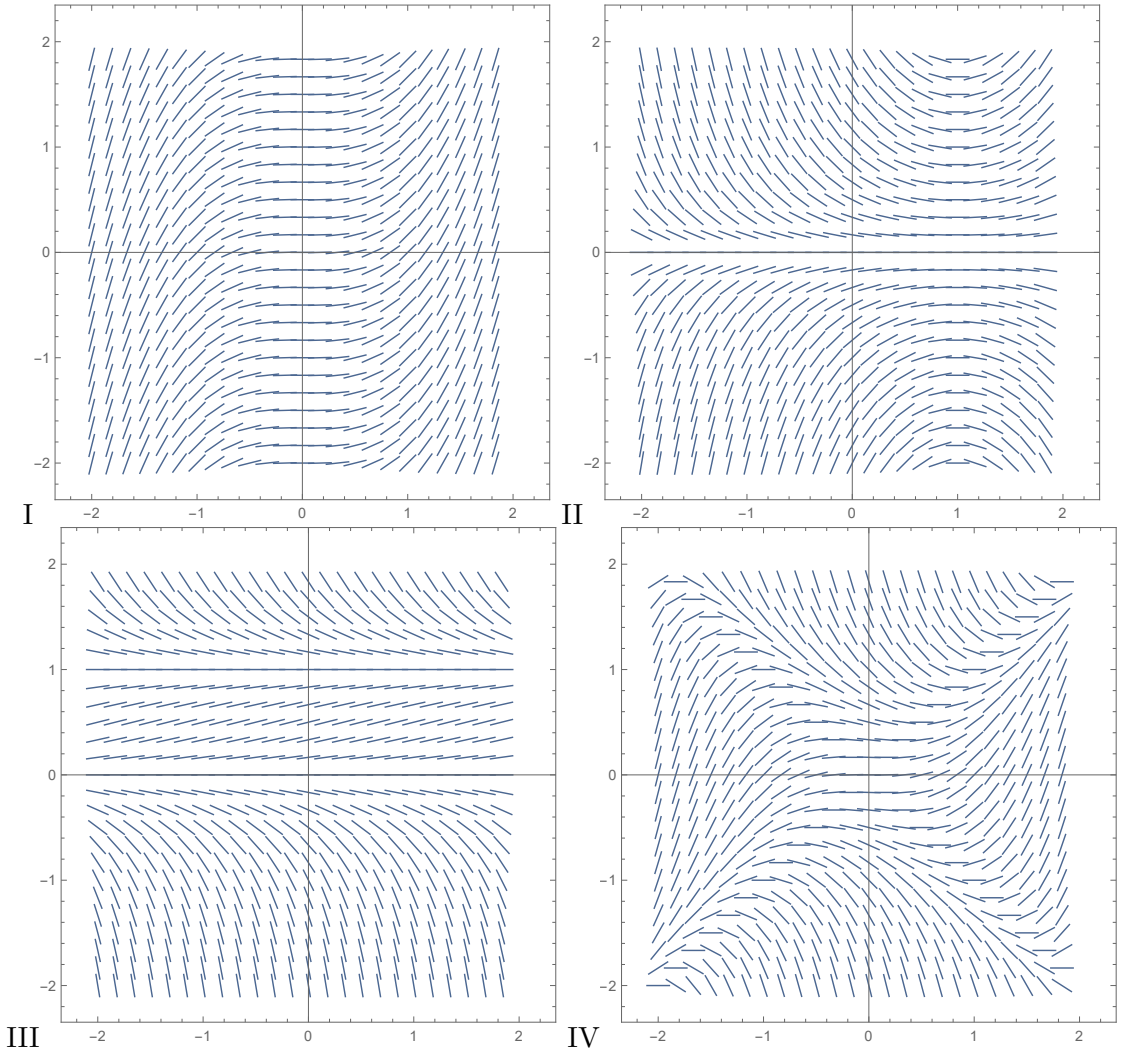
MATH 2300-015 QUIZ 11 (in class) Name: _____

1. Match the slope fields below (labeled I, II, III, IV) with the differential equations below.



_____ $\dfrac{dy}{dx} = x^2$

_____ $\dfrac{dy}{dx} = y - y^2$

_____ $\dfrac{dy}{dx} = x^2 - y^2$

_____ $\dfrac{dy}{dx} = xy - y$

2. Use Euler's method with step size $1/2$ to approximate $y(2)$ where y is a solution of the initial value problem

$$y' = x - y,\ y(0) = 1,$$

filling in the information in the table below.

n	x_n	y_n	$y'(x_n)$
0			
1			
2			
3			
4			

3. Using Taylor's inequality, show that the nth degree Taylor polynomial for $\cos x$ converges to $\cos x$ as $n \to \infty$, i.e. show that $\cos x$ is equal to its Taylor series for all x.

MATH 2300-015 QUIZ 11 Due Tuesday, November 14th Name: _____

1. In this problem, you will show that Euler's method converges to an actual solution of the initial value problem below as you take smaller and smaller step sizes.

   (a) Use Euler's method to obtain an estimate $E_n(x)$ of the solution to

   $$y' = y,\ y(0) = 1,$$

   at x by breaking up the interval between 0 and x into n equal pieces.

   (b) Find the limit as n approaches infinity in your previous answer, i.e. find

   $$E(x) := \lim_{n\to\infty} E_n(x).$$

   (c) Show that the limit $E(x)$ above satisfies the initial value problem.

2. Solve the following initial value problem using power series

   $$y'' + y = 0,\ y(0) = 0,\ y'(0) = 1,$$

   i.e. assume $y = \sum_{n=0}^{\infty} c_n x^n$ is a solution and solve for the c_n recursively.