MATH 2300-015 QUIZ 10 (in class) Name: ____________________________

1. What are the possible intervals of convergence for a general power series $\sum_{n=0}^{\infty} c_n(x-a)^n$?

   [There are six possibilities depending on the radius of convergence.]

2. Given a function $f(x)$ that is infinitely differentiable at $x = a$, what is its Taylor series centered at a? [I.e., how do the coefficients depend on f?]

3. [Memorization] What are the Taylor series for the following functions (centered at zero)?

   (a) $\sin x$

   (b) $\cos x$

   (c) e^x

   (d) $\dfrac{1}{1-x}$

   (e) $\ln(1+x)$

MATH 2300-015 QUIZ 10 Due Tuesday, November 7th Name: ____________________

1. For this problem, let $f(x) = (1+x)^{1/3}$

   (a) Find $f'(x)$, $f''(x)$, and $f'''(x)$.

   (b) What is the maximum M of $|f'''(x)|$ on the interval $[0,1]$?

   (c) What is $T_2(x)$, the second degree Taylor polynomial for f centered at $x=0$?

   (d) Use $T_2(x)$ to estimate $\sqrt[3]{2}$.

   (e) Bound the absolute value of the remainder $R_2(1) = f(1) - T_2(1) = \sqrt[3]{2} - T_2(1)$ using Taylor's inequality and the bound M on $|f'''(x)|$ you found above.

2. (a) Find $\lim_{x \to 0} \dfrac{1 - x^2 - e^{-x^2}}{x^4}$ (using a power series representation for e^{-x^2}).

   (b) Find

$$\int_0^1 \frac{1 - x^2 - e^{-x^2}}{x^4} dx$$

   by integrating a power series term-by-term (your answer will be an infinite series).

3. Find the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n n}{\sqrt{n^3+1}}(x-1)^n$