MATH 2300-005 QUIZ 4 Name: ______________________

Due Monday, September 19th at the beginning of class. Please use additional paper as necessary to submit CLEAR and COMPLETE solutions.

1. Consider the region R in the xy-plane bounded by the curves

$$y = 0,\ x = e,\ y = \ln x.$$

Find the volumes of the solids obtained by rotating R about the coordinate axes.

Rotating about the x-axis, we get (integrating by parts twice)

$$\pi \int_1^e (\ln x)^2 dx = \pi \left[x(\ln x)^2 \Big|_1^e - 2\int_1^e \ln x \ dx \right] = \pi \left[x(\ln x)^2 - 2(x\ln x - x) \right]_1^e$$
$$= \pi(e-2).$$

Rotating about the y-axis, we get

$$\pi \int_0^1 [(e)^2 - (e^y)^2] dy = \pi \left[e^2 y - \frac{e^{2y}}{2} \right]_0^1 = \frac{\pi}{2}(e^2+1).$$

2. Find the volume of the solid obtained by rotating the region between the curves

$$y = \sin x,\ y = \cos x,\ \frac{\pi}{4} \le x \le \frac{5\pi}{4}$$

around the line $y = -1$.

The integral is

$$\pi \int_{\pi/4}^{5\pi/4} \left[(1+\sin x)^2 - (1+\cos x)^2 \right] dx = \pi \int_{\pi/4}^{5\pi/4} \left[2\sin x + \sin^2 x - 2\cos x - \cos^2 x \right] dx$$
$$= \pi \int_{\pi/4}^{5\pi/4} \left[2(\sin x - \cos x) - \cos 2x \right] dx = -\pi \left[2(\sin x + \cos x) + \frac{\sin 2x}{2} \right]_{\pi/4}^{5\pi/4}$$
$$= \pi 4\sqrt{2}.$$

3. Find the volume of the solid obtained by rotating the region between the curves

$$y = 0,\ x = 1,\ y = 1/x$$

about the x-axis. [Note that this is an improper integral.]

The integral is

$$\int_1^\infty \frac{\pi}{x^2} dx = \lim_{t\to\infty} \int_1^t \frac{\pi}{x^2} dx = \lim_{t\to\infty} -\frac{\pi}{x}\Big|_1^t$$
$$= \lim_{t\to\infty} -\frac{\pi}{t} + \pi = \pi.$$

4. Find the volume of the solid obtained by rotating the region inside the circle $(x-R)^2+y^2=r^2$ around the y-axis (assume $R>r$). [You should get $2\pi^2r^2R$.]

   The integral is

$$\pi\int_{-r}^{r}\left[\left(R+\sqrt{r^2-y^2}\right)^2-\left(R-\sqrt{r^2-y^2}\right)^2\right]dy=8R\pi\int_0^r\sqrt{r^2-y^2}dy$$
$$=2\pi^2r^2R,$$

   recognizing the last integral as the area of a quarter-disk with radius r, or you can integrate using an inverse trigonometric substitution ($y=r\sin\theta$).