MATH 2300-005 QUIZ 3 Name: ____________________________

Due Tuesday, September 13th at the beginning of class. Please use additional paper as necessary to submit CLEAR and COMPLETE solutions.

1. We know that $\int_1^\infty \frac{dx}{x}$ diverges but increasing the exponent of x in the denominator by any amount produces a convergent improper integral. Show that the family of functions $\{x(\ln x)^p : p > 0\}$ is "between" x and the family $\{x^p : p > 1\}$ in the following sense:
$$\lim_{x\to\infty} \frac{x(\ln x)^p}{x^q} = 0, \quad \text{for any } q > 1, p > 0.$$
(If you find this too confusing, you may do only the cases $p = 1, 2, 3$.)

2. For which values of p does $\int_e^\infty \frac{dx}{x(\ln x)^p}$ converge/diverge? Find the value of the improper integral when it is convergent.

3. For what values of p does the improper integral $\int_0^1 \frac{dx}{x^p}$ converge?

4. First, show that $\int_0^\infty \frac{dx}{x^3+1}$ converges by comparison. Second, find the value of the improper integral (you should get $\frac{2\pi}{3\sqrt{3}}$).

5. Find the value of C for which the following improper integral converges and evaluate the integral for this value of C:
$$\int_0^\infty \left(\frac{1}{\sqrt{x^2+4}} - \frac{C}{x+2} \right) dx.$$