MATH 2300-005 QUIZ 1 Name: ______________________________

Due Monday, August 29th at the beginning of class.

Use integration by parts to find the following:

- $\displaystyle\int_4^9 \frac{\ln y}{\sqrt{y}}dy$. With $u = \ln y$, $dv = y^{-1/2}dy$, we have $du = \frac{dy}{y}$, $v = 2y^{1/2}$ and the integral becomes

$$2\sqrt{y}\ln y\Big|_4^9 - \int_4^9 2\sqrt{y}\frac{1}{y}dy = 2\sqrt{y}\ln y - 4\sqrt{y}\Big|_4^9 = (6\ln 9 - 12) - (4\ln 4 - 8) = \ln\left(\frac{531441}{256}\right) - 4.$$

- $\displaystyle\int \frac{x}{e^{2x}}dx$. With $u = x$, $dv = e^{-2x}dx$, we have $du = dx$, $v = -\frac{1}{2}e^{-2x}$ and the integral becomes

$$-\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}dx = -\frac{e^{-2x}}{2}(x + 1/2).$$

- $\displaystyle\int t^2\sin(\pi t)dt$. With $u = t^2$, $dv = \sin(\pi t)dt$, we have $du = 2tdt$, $v = -\frac{1}{\pi}\cos(\pi t)$ and the integral becomes

$$-\frac{t^2}{\pi}\cos(\pi t) + \frac{2}{\pi}\int t\cos(\pi t)dt.$$

Using integration by parts again on the new integral ($u = t$, $dv = \cos(\pi t)dt$, $du = dt$, $v = \sin(\pi t)/\pi$) we get

$$-\frac{t^2}{\pi}\cos(\pi t) + \frac{2}{\pi}\left[\frac{t}{\pi}\sin(\pi t) - \frac{1}{\pi}\int \sin(\pi t)dt\right] = -\frac{t^2}{\pi}\cos(\pi t) + \frac{2t}{\pi^2}\sin(\pi t) + \frac{2}{\pi^3}\cos(\pi t).$$

- $\displaystyle\int \cos(\sqrt{x})dx$ (Make a substitution first.) With the substitution

$$y = \sqrt{x},\ dy = \frac{dx}{2\sqrt{x}},\ dx = 2\sqrt{x}dy = 2ydy,$$

the integral becomes $2\displaystyle\int y\cos y\ dy$. Using integration by parts with $u = y$, $dv = \cos y\ dy$, $du = dy$, $v = \sin y$, this integral is

$$2y\sin y - 2\int \sin y\ dy = 2y\sin y + 2\cos y = 2\sqrt{x}\sin(\sqrt{x}) + 2\cos(\sqrt{x}).$$

- $\int e^{-x}\cos(\pi x)dx$. With $u = e^{-x}$, $dv = \cos(\pi x)dx$, we have $du = -e^{-x}dx$, $v = \frac{1}{\pi}\sin(\pi x)$ and the integral becomes
$$\frac{e^{-x}}{\pi}\sin(\pi x) + \frac{1}{\pi}\int e^{-x}\sin(\pi x)dx.$$
Integrating by parts again allows us to solve algebraically for the integral with which we started. Letting $u = e^{-x}$, $dv = \sin(\pi x)dx$, $du = -e^{-x}$, $v = -\cos(\pi x)/\pi$ we have
$$\int e^{-x}\cos(\pi x)dx = \frac{e^{-x}}{\pi}\sin(\pi x) + \frac{1}{\pi}\left[-\frac{e^{-x}}{\pi}\cos(\pi x) - \int e^{-x}\cos(\pi x)dx\right],$$
so that
$$\int e^{-x}\cos(\pi x)dx = \frac{e^{-x}}{1+\pi}(\sin(\pi x) - \cos(\pi x)/\pi).$$

- $\int (\ln x)^2\ dx$. With $u = (\ln x)^2$, $dv = dx$, $du = 2\ln x\frac{dx}{x}$, $v = x$, the integral is
$$x(\ln x)^2 - 2\int \ln x\ dx = x(\ln x)^2 - 2(x\ln x - x),$$
the second equality coming from memorization or integration by parts again.

- $\int \arccos z\ dz$. Trying $u = \arccos z$, $dv = dz$, we get $du = \frac{-dz}{\sqrt{1-z^2}}$, $v = z$ and the integral becomes
$$z\arccos z + \int \frac{zdz}{\sqrt{1-z^2}}.$$
We now make a substitution, $u = 1 - z^2$, $du = -2zdz$ to get
$$z\arccos z - \frac{1}{2}\int u^{-1/2}du = z\arccos z - \sqrt{1-z^2}.$$

- $\int \sin(\ln w)dw$ (Make a substitution first.) With the substitution
$$z = \ln w,\ dz = dw/w,\ dw = wdz = e^z dz,$$
the integral becomes
$$\int e^z \sin z\ dz = \frac{e^z}{2}(\sin z - \cos z),$$
the second equality coming from the trick used above (integration by parts twice, solve algebraically for the integral).

- $\int_1^{\sqrt{3}} \arctan(1/x)dx$. With $u = \arctan(1/x)$, $dv = dx$, $du = \frac{-dx}{1+x^2}$, $v = x$, the integral becomes
$$x\arctan(1/x)\Big|_1^{\sqrt{3}} + \int \frac{xdx}{1+x^2}.$$
Now make the substitution $u = 1 + x^2$, $du = 2xdx$, to get
$$x\arctan(1/x)\Big|_1^{\sqrt{3}} + \frac{1}{2}\int_2^4 \frac{du}{u} = \sqrt{3}\arctan(1/\sqrt{3}) - \arctan(1) + \frac{1}{2}(\ln 2) = \frac{\pi}{2\sqrt{3}} - \frac{\pi}{4} + \frac{1}{2}\ln 2.$$

- $\int x^3\sqrt{1+x^2}\ dx$. [*Solution 1.*] With $u = x^2$, $dv = x\sqrt{1+x^2}$, $du = 2xdx$, $v = \int x\sqrt{1+x^2}dx = \frac{1}{3}(1+x^2)^{3/2}$ (using a substitution), we get

$$\frac{x^2}{3}(1+x^2)^{3/2} - \frac{1}{3}\int 2x(1+x^2)^{3/2}\ dx = \frac{x^2}{3}(1+x^2)^{3/2} - \frac{2}{15}(1+x^2)^{5/2},$$

using a substitution in the second integral. [*Solution 2.*] Instead of doing integration by parts, one can directly make the substitution

$$u = 1+x^2,\ du = 2xdx,\ x^2 = u-1,$$

to get

$$\frac{1}{2}\int (u-1)\sqrt{u}\ du = \frac{u^{5/2}}{5} - \frac{u^{3/2}}{3} = \frac{1}{5}(1+x^2)^{5/2} - \frac{1}{3}(1+x^2)^{3/2}.$$

The solutions don't immediately look the same, but they are both equal to

$$(1+x^2)^{3/2}\frac{3x^2-2}{15}.$$

- $\int \sec^3\theta\ d\theta$. With $u = \sec\theta$, $dv = \sec^2\theta\ d\theta$, $du = \sec\theta\tan\theta\ d\theta$, $v = \tan\theta$, we get

$$\sec\theta\tan\theta - \int \tan^2\theta\sec\theta\ d\theta.$$

Using the Pythagorean identity $1+\tan^2\theta = \sec^2\theta$ in the integrand, we get

$$\int \sec^3\theta\ d\theta = \sec\theta\tan\theta - \int(\sec^2\theta - 1)\sec\theta\ d\theta = \sec\theta\tan\theta - \int \sec^3\theta\ d\theta + \int \sec\theta\ d\theta.$$

Solving algebraically for $\int \sec^3\theta\ d\theta$ and recalling

$$\int \sec\theta\ d\theta = \int \sec\theta\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta} = \ln|\sec\theta+\tan\theta|$$

(using the substitution $u = \sec\theta + \tan\theta$) gives

$$\int \sec^3\theta\ d\theta = \frac{1}{2}[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta|].$$

- Show that

$$\int(\ln x)^n dx = x(\ln x)^n - n\int(\ln x)^{n-1}dx.$$

With $u = (\ln x)^n$, $dv = dx$, $du = n(\ln x)^{n-1}\frac{dx}{x}$, $v = x$, we get

$$x(\ln x)^n - n\int(\ln x)^{n-1}dx.$$