

Due Monday, November 28th at the beginning of class.

1. Recall Newton's law of cooling: the rate of change in temperature of an object is proportional to the difference in temperature between the object and its surroundings,

$$\frac{dT}{dt} = k(T - T_s),$$

where $T(t)$ is temperature as a function of time, k is the proportionality constant, and T_s is the constant surrounding temperature.

Suppose a cup of coffee is 200°F when it is poured and has cooled to 190°F after one minute in a room at 70°F . When will the coffee reach 150°F ? What will the temperature of the coffee be after it sits for 30 minutes?

2. Differentiate the logistic equation (k, M constant)

$$\frac{dP}{dt} = kP(1 - P/M)$$

with respect to t to get

$$\frac{d^2P}{dt^2} = k^2P(1 - P/M)(1 - 2P/M).$$

What can you deduce from this about solutions $P(t)$?

3. The following variation on the logistic equation models logistic growth with constant harvesting:

$$\frac{dP}{dt} = kP(1 - P/M) - c.$$

For this problem consider the specific instance

$$\frac{dP}{dt} = 0.08P(1 - P/1000) - 15,$$

modeling fish population in a pond where 15 fish per week are caught (time t in weeks).

- What are the equilibrium solutions to the differential equation in part (i.e. what are the constant solutions)?
 - Find the general solution of the differential equation. [Integrate using partial fractions. You should get $P(t) = \frac{750 - 250Ce^{-t/25}}{1 - Ce^{-t/25}}$ where C is an arbitrary constant.]
 - Find and interpret the solutions with initial conditions $P(0) = 200, 300$.
4. Here's a "heads-up" concerning material for the rest of the semester. You will definitely be responsible for the material in sections 1.7, 3.4, and 6.4 concerning parametric curves, slopes/tangents to parametric curves, and arclength of parametric curves. [You may also want to look at sections 9.1, 9.2, and 9.5 concerning 3D coordinates and vectors, and 10.1, 10.2, 10.3 concerning some simple vector calculus - this is mostly optional.] We will also be covering the material in appendices H.1 and H.2, polar coordinates and arclength/area in polar coordinates.