

1. Find a power series representation (centered at zero) for

$$\frac{1}{(1+x^3)^2} = \frac{d}{dy} \left(\frac{1}{1-y} \right)_{y=-x^3}$$

2. Solve the following initial value problems.

(a) $y' + y^2 \sin x = 0$, $y(0) = -1/2$

(b) $y' = \frac{x^2}{y(1+x^3)}$, $y(0) = -1$

3. Suppose $y(x)$ is the solution to the initial value problem

$$y' = x^2 - y^2, \quad y(0) = 1.$$

Use Euler's method (step size 0.1) to approximate $y(0.5)$.

4. Use the third degree Taylor polynomial (centered at zero) for $f(x) = \ln(1+x)$ to estimate $\ln(2)$ and use Taylor's inequality to give bounds on the error.