MATH 2300-005 QUIZ 10 Name: ___________________________

This quiz is due Tuesday, November 1st at the beginning of class. Use additional paper as necessary to submit CLEAR and COMPLETE solutions.

1. Find $\lim_{x \to 0} \dfrac{1 - x^2 - e^{-x^2}}{x^4}$ (using a power series representation for e^{-x^2}). [You may assume e^x is equal to its Taylor series.]

2. Find
$$\int_0^1 \frac{1 - x^2 - e^{-x^2}}{x^4} dx$$
by integrating a power series term-by-term.

3. Use the alternating series remainder estimate to to give an approximation to the above integral so that the error is less than 0.001.

4. Find the Taylor series centered at zero for the function $f(x) = (1 - x^2)^{-1/2}$. [You may assume the results of the text on the binomial series, cf. §8.7, pg. 611–612).

5. Integrate the power series from the previous problem term-by-term to find a power series representation for $\arcsin x$ around $x = 0$.

6. (Optional "fun" problem) Suppose
$$f(x) = \frac{1}{1 - x - x^2} = \sum_{n=0}^{\infty} c_n x^n.$$
Multiply both sides by $1 - x - x^2$ and equate powers of x to show that $c_n = F_n$, the nth Fibonacci number ($F_0 = 1, F_1 = 1, F_n = F_{n-1} + F_{n-2}$). Use partial fractions to write
$$f(x) = \frac{A}{x + \phi} + \frac{B}{x + \bar{\phi}}$$
where $\phi = \frac{1+\sqrt{5}}{2}$, $\bar{\phi} = \frac{1-\sqrt{5}}{2}$. Write $f(x)$ as a power series using the geometric series and the partial fraction decomposition. Finally, give a closed-form expression for the F_n. [You should get something equivalent to $F_n = \dfrac{(-1)^n}{\sqrt{5}} \left(\dfrac{1}{\phi^{n+1}} - \dfrac{1}{\bar{\phi}^{n+1}} \right) = \dfrac{\phi^{n+1} - \bar{\phi}^{n+1}}{\sqrt{5}}$.]