Quiz 4, MATH 1300-401

3.2/25

$$\begin{aligned} y &= x^4e^x \\ y' &= x^4(e^x)' + e^x(x^4)' = x^4e^x + 4x^3e^x = e^x(x^4+4x^3) = x^3e^x(x+4) \\ y'' &= (e^x(x^4+4x^3))' = e^x(x^4+4x^3)' + (x^4+4x^3)(e^x)' = e^x(4x^3+12x^2) + (x^4+4x^3)e^x \\ &= x^2e^x(x^2+8x+12) \end{aligned}$$

3.2/26

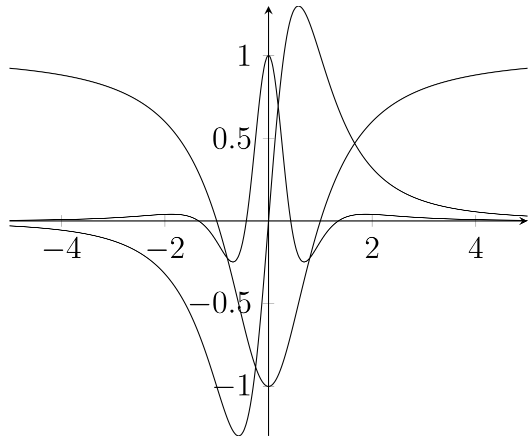
$$\begin{aligned} y &= x^{5/2}e^x \\ y' &= \frac{5}{2}x^{3/2} + e^x x^{5/2} = e^x\left(\frac{5}{2}x^{3/2} + x^{5/2}\right) = x^{3/2}e^x\left(x + \frac{5}{2}\right) \\ y'' &= \left(e^x\left(\frac{5}{2}x^{3/2} + x^{5/2}\right)\right)' = e^x\left(\frac{5}{2}x^{3/2} + x^{5/2}\right) + e^x\left(\frac{15}{4}x^{1/2} + \frac{5}{2}x^{3/2}\right) \\ &= x^{1/2}e^x\left(\frac{15}{4} + 5x + x^2\right) \end{aligned}$$

3.2/27

$$\begin{aligned} f(x) &= \frac{x^2}{1+2x} \\ f'(x) &= \frac{(1+2x)(x^2)' - (x^2)(1+2x)'}{(1+2x)^2} = \frac{(1+2x)2x - (x^2)2}{(1+2x)^2} \\ &= \frac{2x^2+2x}{(1+2x)^2} \\ f''(x) &= \frac{(1+2x)^2(2x^2+2x)' - (2x^2+2x)((1+2x)^2)'}{((1+2x)^2)^2} \\ &= \frac{(1+2x)^2(4x+2) - (2x^2+2x)(4(1+2x))}{(1+2x)^4} \\ &= \frac{2}{(1+2x)^3} \end{aligned}$$

???

$$\begin{aligned} f(x) &= \frac{x^2-1}{x^2+1} \\ f'(x) &= \frac{(x^2+1)(x^2-1)' - (x^2-1)(x^2+1)'}{(x^2+1)^2} = \frac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2} \\ &= \frac{4x}{x^4+2x^2+1} \\ f''(x) &= \frac{(x^4+2x^2+1)(4x)' - (4x)(x^4+2x^2+1)'}{(x^4+2x^2+1)^2} = \frac{-4(3x^4-6x^2+1)}{(x^2+1)^4} \\ &= \frac{-12(x^2-(1+\sqrt{2/3}))(x^2-(1-\sqrt{2/3}))}{(x^2+1)^4} \text{ (etc., needless factoring at this point)} \end{aligned}$$



3.3/10

$$\begin{aligned} y &= \frac{1+\sin x}{x+\cos x} \\ y' &= \frac{(x+\cos x)(1+\sin x)' - (1+\sin x)(x+\cos x)'}{(x+\cos x)^2} \\ &= \frac{(x+\cos x)(\cos x) - (1+\sin x)(1-\sin x)}{(x+\cos x)^2} \\ &= \frac{x\cos x}{(x+\cos x)^2} \text{ (using } \cos^2+\sin^2=1) \end{aligned}$$

3.3/13

$$\begin{aligned} f(x) &= xe^x \csc x \\ f'(x) &= (xe^x)(\csc x)' + (\csc x)(xe^x)' = -xe^x \csc x \cot x + \csc x(xe^x + e^x) \\ &= e^x \csc x(x+1-x\cot x) \end{aligned}$$

3.3/14

$$\begin{aligned} y &= x^2 \sin x \tan x \\ y' &= (x^2)(\sin x \tan x)' + (\sin x \tan x)(x^2)' = x^2(\sin x \sec^2 x + \tan x \cos x) + 2x \sin x \tan x \\ &= x\tan x(\sec x + \cos x + \sin x) \text{ (needless simplification)} \end{aligned}$$

3.3/41 Find a solution of the form $y = A\sin x + B\cos x$ to the differential equation

$$y'' + y' - 2y = \sin x.$$

We have

$$\begin{aligned} y' &= A\cos x - B\sin x \\ y'' &= -A\sin x - B\cos x. \end{aligned}$$

Plugging this into the differential equation we get

$$\begin{aligned}\sin x &= [-A\sin x - B\cos x] + [A\cos x - B\sin x] - 2[A\sin x + B\cos x]\\ \sin x &= (-3A - B)\sin x + (-3B + A)\cos x\end{aligned}$$

so that (equating coefficients of $\sin x, \cos x$ on both sides)

$$-3A - B = 1,\ -3B + A = 0$$

with the solution $A = -3/10,\ B = -1/10$.

3.4/18

$$\begin{aligned}y &= e^{-2t}\cos(4t)\\ y' &= -4e^{-2t}\sin(4t) - 2e^{-2t}\cos(4t) = -2e^{-2t}(\cos(4t) + 2\sin(4t))\end{aligned}$$

3.4/20

$$\begin{aligned}h(t) &= (t^4-1)^3(t^3+1)^4\\ h'(t) &= 4(t^3+1)^3(3t^2)(t^4-1)^3 + 3(t^4-1)^2(4t^3)(t^3+1)^4\\ &= 12t^2(t^3+1)^3(t^4-1)^3(2t^4+t-1)\end{aligned}$$

3.4/21

$$\begin{aligned}y &= e^{x\cos x}\\ y' &= e^{x\cos x}(-x\sin x + \cos x)\end{aligned}$$

3.4/23

$$\begin{aligned}y &= \left(\frac{x^2+1}{x^2-1}\right)^3\\ y' &= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}\\ &= \frac{-4x}{(x^2-1)^2}\end{aligned}$$

3.4/31

$$\begin{aligned}y &= 2^{\sin \pi x}\\ y' &= 2^{\sin \pi x}\ln 2(\pi\cos \pi x)\end{aligned}$$

3.4/32

$$\begin{aligned}y &= \sin(\sin(\sin x))\\ y' &= \cos(\sin(\sin x))\cos(\sin x)\cos x\end{aligned}$$

3.4/34

$$
\begin{aligned}
y &= \sqrt{x+\sqrt{x+\sqrt{x}}} \\
y' &= \frac{1}{2}\left(x+\sqrt{x+\sqrt{x}}\right)^{-1/2}\left(1+\frac{1}{2}\left(x+\sqrt{x}\right)^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)\right) \\
&= \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left(1+\frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right)
\end{aligned}
$$

3.4/49 Find all points on the graph of the function $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

We need to solve

$$f'(x) = 2\cos x + 2\sin x\cos x = 2\cos x(1+\sin x) = 0.$$

This happens when $\cos x = 0$ (odd multiples of $\pi/2$, $x = (2k+1)\frac{\pi}{2}$) or when $\sin x = -1$ $(3\pi/2 + 2\pi k)$. The corresponding y-values are $3, -1$, (depending on whether x is $\pi/2$ or $3\pi/2$ up to a multiple of 2π).