Quiz 3, MATH 1300-401

2.7/3 a II, b IV, c I, d III

2.7/42 a f''', b f'', c f', d f

2.7/50 Consider $g(x) = x^{2/3}$. Trying to compute $g'(0)$, we get

$$g'(0) = \lim_{h\to 0} \frac{h^{2/3} - 0^{2/3}}{h} = \lim_{h\to 0} h^{-1/3},$$

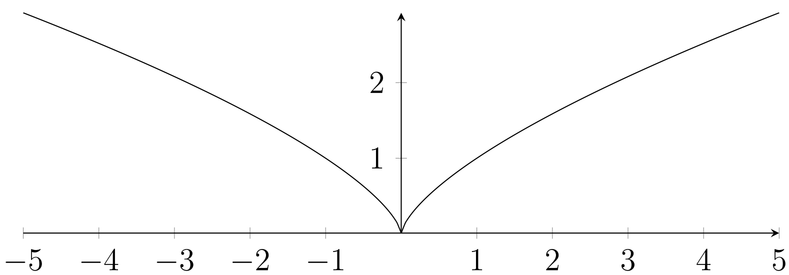
which does not exist. We have

$$\lim_{h\to 0^+} h^{-1/3} = +\infty, \lim_{h\to 0^-} h^{-1/3} = -\infty.$$

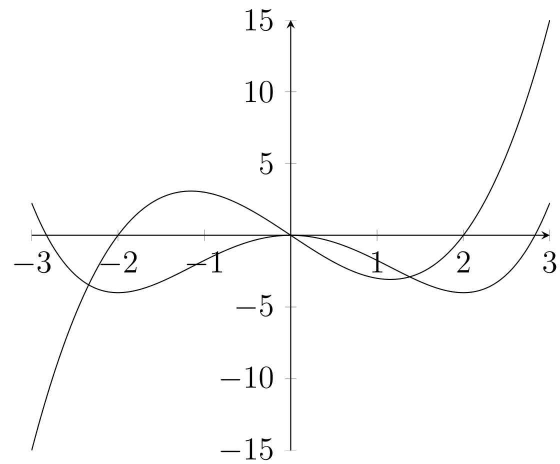
The function g is differentiable everywhere else (recall that $a^3-b^3 = (a-b)(a^2+ab+b^2)$)

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{(x+h)^{2/3} - x^{2/3}}{h} = \lim_{h\to 0} \frac{(x+h)^{2/3} - x^{2/3}}{h} \frac{(x+h)^{4/3} + (x+h)^{2/3}x^{2/3} + x^{4/3}}{(x+h)^{4/3} + (x+h)^{2/3}x^{2/3} + x^{4/3}} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h((x+h)^{4/3} + (x+h)^{2/3}x^{2/3} + x^{4/3})} = \lim_{h\to 0} \frac{2x+h}{(x+h)^{4/3} + (x+h)^{2/3}x^{2/3} + x^{4/3}} \\ &= \frac{2x}{3x^{4/3}} = \frac{2}{3}x^{-1/3} \end{aligned}$$

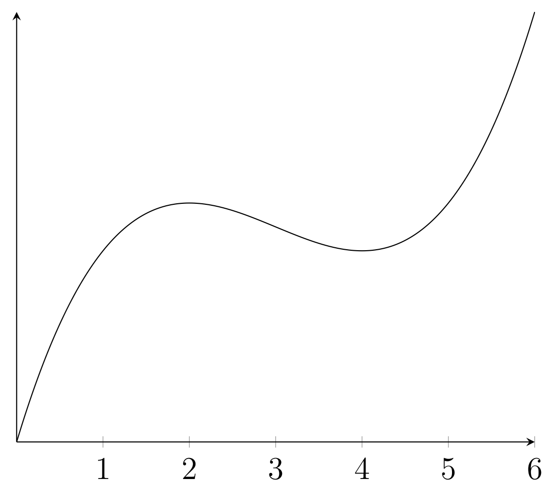
(i.e. the power rule with exponent 2/3). Below in a graph of g (note the "cusp" at $x = 0$).



2.8/2 f is increasing on $(-2, 0) \cup (2, \infty)$ and decreasing on $(-\infty, -2) \cup (0, 2)$, with local minima at $x = -2, 2$ and a local maximum at $x = 0$.



2.8/12 The position $s(t)$ of a particle is shown ($+/-$ position indicates left/right). The particle is moving to the right on $(0,2) \cup (4,\infty)$ and to the left on $(2,4)$ (corresponding to positive/negative velocity $s'(t)$, i.e. increasing/decreasing s). The particle is accelerating on $(3,\infty)$ and decelerating on $(0,3)$ (corresponding to positive/negative aceleration $s''(t)$, i.e. concavity of s).



2.8/29 Graph b is the antiderivative of f (graph a is the derivative of f).

3.1/31 If $y = 3x^2 - x^3$, then $y' = 6x - 3x^2$ (and $y'(1) = 3$). The tangent line through $(1,2)$ is given by $y - 2 = 3(x-1)$

3.1/32 If $y = x - \sqrt{x}$, then $y' = 1 - \frac{1}{2}x^{-1/2}$ (and $y'(1) = 1/2$). The tangent line through $(1,0)$ is given by $y - 0 = (1/2)(x-1)$

3.1/49 Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent line is horizontal.

We have $y' = 6x^2 + 6x - 12 = 6(x+2)(x-1)$ which is zero when $x = -2, 1$ with $y(-2) = 21, y(1) = -6$. So the points on the graph where horizontal tangent lines occur are $(-2, 21), (1, -6)$.

3.1/61 Find a quadratic polynomial P such that $P(2) = 5, P'(2) = 3, P''(2) = 2$.

Start with $P(x) = ax^2 + bx + c, P'(x) = 2ax + b, P''(x) = 2a$. We want

$$P(2) = 5 = 4a + 2b + c, P'(2) = 3 = 4a + b, P''(2) = 2 = 2a.$$

Solving this system we get $a = 1, b = -1, c = 3$.