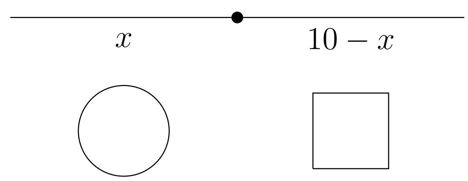
Math 1300-001, Quiz 8 Name: ________________________

1. A length of wire 10 meters in length is cut into two lengths, x and $10-x$. The first is made into a circle and the second into a square. For what value of x is the total area of the two figures minimized? (Recall: the circumference of a circle is $2\pi r$ and the area of a circle is πr^2 where r is the radius of the circle.) Answer: $\frac{10\pi}{4+\pi}$.



The circumference of the circle is $2\pi r = x$ so that the area of the circle as a function of x is

$$A_{\text{circ}}(x) = \pi r^2 = \pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}.$$

The perimeter of the square is $10-x$ so that the side length is $(10-x)/4$ and the area of the square as a function of x is

$$A_{\text{sq}}(x) = \left(\frac{10-x}{4}\right)^2.$$

Hence the total area of the two figures, as a function of x is

$$A(x) = A_{\text{circ}}(x) + A_{\text{sq}}(x) = \frac{x^2}{4\pi} + \left(\frac{10-x}{4}\right)^2, \ x \in [0,10].$$

We differentiate $A(x)$ and look for critical numbers

$$A'(x) = \frac{x}{2\pi} + \frac{x-10}{8}, \ A'(x) = 0 \Rightarrow x = \frac{10\pi}{4+\pi} =: x_{\text{min}}.$$

Because $A(x)$ is decreasing on $[0, x_{\text{min}}]$ and increasing on $[x_{\text{min}}, 10]$, $A(x_{\text{min}}) \approx 3.5$ is actually the minimum value of A on $[0,10]$.

2. Use l'Hôpital's rule to find the following limits.

(a) $\displaystyle\lim_{t\to 0} \frac{e^t - 1 - t}{t^2}$

This limit is of the indeterminate form "0/0" so that we can apply l'Hôpital's rule:

$$\lim_{t\to 0} \frac{e^t - 1 - t}{t^2} \underset{\text{l'H}}{=} \lim_{t\to 0} \frac{e^t - 1}{2t} = \frac{0}{0}.$$

The limit we obtain is also of the form "0/0", so we can apply l'Hôpital's rule again:

$$\lim_{t\to 0} \frac{e^t - 1 - t}{t^2} \underset{\text{l'H}}{=} \lim_{t\to 0} \frac{e^t - 1}{2t} \underset{\text{l'H}}{=} = \lim_{t\to 0} \frac{e^t}{2} = \frac{1}{2}.$$

(b) $\displaystyle\lim_{x\to\infty} xe^{1/x} - x$

As it stands, this limit is in the indeterminate form "$\infty - \infty$". To apply l'Hôpital's rule we must be in the form "0/0" or "∞/∞". Factor out the x and change multiplication by x to division by $1/x$ to obtain

$$\lim_{x\to\infty} xe^{1/x} - x = \lim_{x\to\infty} x(e^{1/x} - 1) = \lim_{x\to\infty} \frac{e^{1/x} - 1}{1/x} = \frac{0}{0}.$$

Now we can apply l'Hôpital's rule:

$$\lim_{x\to\infty} \frac{e^{1/x} - 1}{1/x} \underset{\text{l'H}}{=} \lim_{x\to\infty} \frac{-x^{-2}e^{1/x}}{-x^{-2}} = \lim_{x\to\infty} e^{1/x} = e^0 = 1.$$