Math 1300-001, Quiz 7 Name: ________________________

Let $f(x) = \dfrac{\ln x}{x^2}$.

1. Some basic properties of f:

   (a) What is the domain of f?
   The domain of f is $(0, \infty)$.

   (b) For what values of x is $f(x) = 0$?

   $$f(x) = 0 \Rightarrow \ln x = 0 \Rightarrow x = 1$$

   (c) What is $\displaystyle\lim_{x \to 0^+} f(x)$?
   $\displaystyle\lim_{x \to 0^+} f(x) = -\infty$

   (d) What is $\displaystyle\lim_{x \to \infty} f(x)$? (Use l'Hôpital's rule.)

   $$\lim_{x \to \infty} \frac{\ln x}{x^2} \underset{\text{l'Hôpital}}{=} \lim_{x \to \infty} \frac{1/x}{2x} = 0$$

2. Find the first and second derivatives of f with respect to x.

   $$f'(x) = \frac{x^2(1/x) - (\ln x)(2x)}{x^4} = \frac{1 - 2\ln x}{x^3}$$

   $$f''(x) = \frac{x^3(-2/x) - (1 - 2\ln x)(3x^2)}{x^6} = \frac{-5 + 6\ln x}{x^4}$$

3. What are the critical numbers of f? Of f'? [Recall that a critical number of a function $g(x)$ is a value of x in the domain of g for which $g'(x)$ is either zero or does not exist.]
   f' is defined on $(0, \infty)$ and

   $$f'(x) = 0 \Rightarrow 1 - 2\ln x = 0 \Rightarrow x = e^{1/2}.$$

   f'' is defined on $(0, \infty)$ and

   $$f''(x) = 0 \Rightarrow -5 + 6\ln x = 0 \Rightarrow x = e^{5/6}.$$

4. List the intervals on which f is increasing/decreasing/concave up/concave down. [You may use either a number line or interval notation.]

   Looking at the sign of the first derivative, we see that f is increasing on $(0, e^{1/2})$ and decreasing on $(e^{1/2}, \infty)$.

   Looking at the sign of the second derivative, we see that f is concave up on $(e^{5/6}, \infty)$ and concave down on $(0, e^{5/6})$.

5. List any local extrema of f and the values of x at which they occur.

   The function f has a local max of $\frac{1}{2e}$ at $x = e^{1/2}$ because f is increasing until $x = e^{1/2}$ and decreasing afterwards.

6. List any inflection points for the graph of f.

   The only inflection point is $(e^{5/6}, \frac{5}{6e^{5/3}})$ where the concavity of the graph changes from concave down to concave up.

7. Sketch the graph of f using the above information, labeling local extrema and inflection points.