

Let  $f(x) = \frac{\ln x}{x^2}$ .

1. Some basic properties of  $f$ :
  - (a) What is the domain of  $f$ ?
  - (b) For what values of  $x$  is  $f(x) = 0$ ?
  - (c) What is  $\lim_{x \rightarrow 0^+} f(x)$ ?
  - (d) What is  $\lim_{x \rightarrow \infty} f(x)$ ? (Use l'Hôpital's rule.)
2. Find the first and second derivatives of  $f$  with respect to  $x$ .
3. What are the critical numbers of  $f$ ? Of  $f'$ ? [Recall that a critical number of a function  $g(x)$  is a value of  $x$  in the domain of  $g$  for which  $g'(x)$  is either zero or does not exist.]

4. List the intervals on which  $f$  is increasing/decreasing/concave up/concave down. [You may use either a number line or interval notation.]
5. List any local extrema of  $f$  and the values of  $x$  at which they occur.
6. List any inflection points for the graph of  $f$ .
7. Sketch the graph of  $f$  using the above information, labeling local extrema and inflection points.