Math 1300-001, Quiz 5 Name: ______________________

1. What are the derivatives of the following functions? (You should know all of these!)

   (a) $\ln x$ and $\log_3 x$

   $$(\ln x)' = \frac{1}{x}, \ (\log_3 x)' = \frac{1}{x \ln 3}.$$

   In general

   $$(\log_a x)' = \frac{1}{x \ln a}.$$

   (b) e^x and 3^x

   $$(e^x)' = e^x, \ (3^x)' = 3^x \ln 3.$$

   In general

   $$(a^x)' = a^x \ln a.$$

   (c) $\tan x$ and $\arctan x$

   $$(\tan x)' = \sec^2 x, \ (\arctan x)' = \frac{1}{1+x^2}.$$

2. Find the derivative of

$$y = \sin(\ln(3x^2+2))$$

$$y' = \cos(\ln(3x^2+2))\frac{6x}{3x^2+2}.$$

3. Use implicit differentiation to find the equation of the tangent line to the curve

$$2(x^2+y^2)^2 = 25(x^2-y^2)$$

going through the point $(3,1)$.

Differentiating both sides and assuming y is a differentiable function of x we have

$$4(x^2+y^2)(2x+2yy') = 25(2x-2yy').$$

Solving for y' we have

$$y' = \frac{50x - 8x(x^2+y^2)}{50y + 8y(x^2+y^2)}.$$

At $(x,y) = (3,1)$ we have $y' = -9/13$. The equation of the line through $(3,1)$ with slope $-9/13$ is

$$y - 1 = \frac{-9}{13}(x-3).$$

4. Find the derivative of arccot x as follows:

(a) Differentiate

$$\cot(\operatorname{arccot} x) = x$$

using the chain rule and solve for $\frac{d}{dx}(\operatorname{arccot} x)$ (recall that $\frac{d}{dx}(\cot x) = -\csc^2 x$).

Differentiating

$$\cot(\operatorname{arccot} x) = x$$

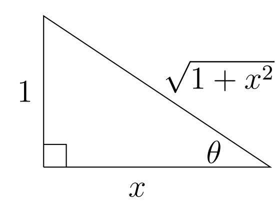
with respect to x gives

$$-\csc^2(\operatorname{arccot} x)\frac{d}{dx}(\operatorname{arccot} x) = 1.$$

Solving for the derivative of arccot x gives

$$\frac{d}{dx}(\operatorname{arccot} x) = \frac{-1}{\csc^2(\operatorname{arccot} x)}.$$

(b) Draw a triangle to write csc(arccot x) as an algebraic function of x (recall that cotangent is adjacent/opposite and cosecant is hypoteneuse/opposite). Consider the picture



where

$$\theta = \operatorname{arccot} x, \ \cot\theta = \frac{\text{adj}}{\text{opp}} = x, \ \csc\theta = \frac{\text{hyp}}{\text{opp}} = \sqrt{1+x^2}.$$

Putting (a) and (b) together, we have

$$\frac{d}{dx}(\operatorname{arccot} x) = \frac{-1}{1+x^2}.$$