Math 1300-001, Quiz 4 Name: ______________________

1. Find the derivatives of the following functions. Do not simplify your solutions.

   (a) $y = \dfrac{3x^3 - x^2 + 1}{5x^2 - 2x + 3}$

   $$y' = (5x^2 - 2x + 3)(9x^2 - 2x) - (3x^3 - x^2 + 1)(10x - 2)(5x^2 - 2x + 3)^2$$

   using the quotient rule, $\left(\dfrac{f}{g}\right)' = \dfrac{gf' - fg'}{g^2}$

   (b) $y = 2^x x^2$

   $$y' = (2^x)'(x^2) + (2^x)(x^2)' = x^2 2^x \ln 2 + 2x2^x = x2^x(x \ln 2 + 2)$$

   using the product rule, $(fg)' = f'g + fg'$ and remembering that $(a^x)' = a^x \ln a$.

   (c) $y = e^{1/x} \sin x$ (use the chain rule to find $\frac{d}{dx} e^{1/x}$)

   $$\begin{aligned} y' &= \left(e^{1/x}\right)' (\sin x) + \left(e^{1/x}\right) (\sin x)' \\ &= \left(-\frac{1}{x^2} e^{1/x}\right) \sin x + e^{1/x} \cos x \\ &= e^{1/x} \left(\cos x - \frac{\sin x}{x^2}\right) \end{aligned}$$

   using the chain rule on $e^{1/x}$, $(e^{1/x})' = e^{1/x}(1/x)' = -e^{1/x}/x^2$.

   (d) $y = \cos(\tan x)$ (use the chain rule)

   $$y' = -\sin(\tan x)(\tan x)' = -\sin(\tan x) \sec^2 x$$

2. Find the following limit:

$$\lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)} \quad \left(\text{recall } \lim_{x \to 0} \frac{\sin x}{x} = 1\right).$$

$$\begin{aligned} \lim_{x \to 0} \frac{\sin(3x)}{\sin(5x)} &= \lim_{x \to 0} \frac{3}{5} \frac{\sin(3x)}{3x} \frac{5x}{\sin(5x)} \\ &= \frac{3}{5} \lim_{x \to 0} \frac{\sin(3x)}{3x} \lim_{x \to 0} \frac{5x}{\sin(5x)} \\ &= \frac{3}{5} \cdot 1 \cdot 1 = \frac{3}{5} \end{aligned}$$

3. Two cars start at the same point at time $t = 0$ (time measure in hours), car 1 heading due east at 40 mph and car 2 heading due south at 30 mph.

   (a) What is the distance between the two cars as a function of time?
   The distance between the two cars is given by

   $$d(t) = \sqrt{(30t)^2 + (40t)^2} = \sqrt{t^2(900 + 1600)} = 50t \text{ miles.}$$

   (b) How fast is the distance between the two cars increasing after an hour?
   We have $d'(t) = 50t$, so $d'(1) = 50$ mph. If you didn't simplify the the expression for $d(t)$, you can differentiate using the chain rule

   $$\begin{aligned} d'(t) &= \frac{1}{2}\left((30t)^2 + (40t)^2\right)^{-1/2}(1800t + 3200t), \\ d'(1) &= \frac{1800 + 3200}{2(\sqrt{900 + 1600})} = 50 \text{ mph.} \end{aligned}$$