This quiz is due Friday, May 1st in class.

1. Find the following limits

    (a) $\lim\limits_{x \to 3} \dfrac{\sqrt{x+6} - x}{x^3 - 3x^2}$

    (b) $\lim\limits_{x \to \pi^-} \ln(\sin x)$

    (c) $\lim\limits_{x \to 0} x^2 \cos(x^{-2})$ (Hint: "squeeze" theorem)

2. Use the intermediate value theorem to show that $x = e^{-x^2}$ has a solution.

3. Use the definition of the derivative (as a limit of averages) to find $\dfrac{d}{dx}\sqrt{1+2x}$. Find the tangent line to the graph of $y = \sqrt{1+2x}$ when $x = 4$.

4. Use the definition of the derivative (as a limit of averages) to find $\dfrac{d}{dx}\dfrac{1}{4x+1}$. Find the tangent to the graph of $y = 1/(4x+1)$ when $x = 4$.

5. Differentiate the following functions

    (a) $\dfrac{e^{1/x}}{x^2}$

    (b) $\dfrac{\sec(2\theta)}{1 + \tan(2\theta)}$

    (c) $3^{x \ln x}$

    (d) $\arctan(\arcsin(\sqrt{x}))$

6. Find dy/dx if x and y are related by

$$x^2 \cos y + \sin(2y) = xy.$$

7. Use a tangent line approximation (to $\sqrt[3]{1+x}$ for instance) to estimate $\sqrt[3]{9}$.

8. Use l'Hôpital's rule to find the following limits

    (a) $\lim\limits_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$

    (b) $\lim\limits_{x \to \infty} \left(1 + \dfrac{a}{x}\right)^{bx}$ $(a, b$ constants$)$

9. Find the dimensions (radius, height) of a cylindrical can with minimal surface area, if it has no top and its volume is 500 cm^3.

10. Water is leaking out of an inverted conical tank at a rate of $10,000 \text{ cm}^3/\text{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which the water is being pumped into the tank.

11. Consider the function
$$f(x) = \frac{x^2 - 16}{x - 5}$$

   (a) What is the domain of f? For what values of x is $f(x) = 0$?

   (b) Find f' and list the intervals on which f is increasing/decreasing.

   (c) Find f'' and list the intervals on which f is concave up/concave down.

   (d) List and classify any local extrema for the function f. Does the graph of f have any inflection points?

   (e) Using the information above, sketch the graph of f.

12. Subdivide the interval $[-3, 5]$ into four equal parts and use a left endpoint Riemann sum to estimate the definite integral

$$\int_{-3}^{5} (x^2 - 1)dx.$$

13. Compute the following definite integrals

   (a) $\displaystyle\int_{1/2}^{3/2} x\sqrt{2x - 1}dx$

   (b) $\displaystyle\int_{0}^{\pi/2} \cos x \sin(\sin x)dx$

   (c) $\displaystyle\int_{1}^{e} \frac{dx}{x\sqrt{\ln x}}$

14. Find the area bounded by the curves

$$y = 2 - x, \ y = x^2.$$