Math 1300-001, Quiz 10 Name: ____________________

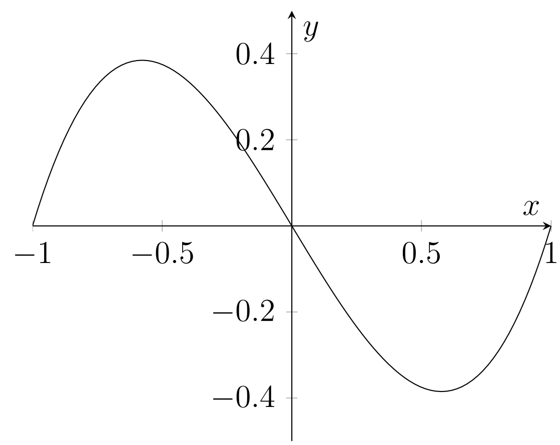
This quiz is due Monday, April 26th in class. SHOW ALL YOUR WORK.

1. Evaluate the following definite integrals

   (a) $\int_{-1}^{1} (x^3 - x)dx$ We have

$$\int_{-1}^{1} (x^3 - x)dx = \frac{x^4}{4} - \frac{x^2}{2}\Big|_{-1}^{1} = \left[\frac{1^4}{4} - \frac{1^2}{2}\right] - \left[\frac{(-1)^4}{4} - \frac{(-1)^2}{2}\right] = 0.$$

   You may also note that the integral is zero because $x^3 - x$ is odd and $[-1, 1]$ is symmetric about the y-axis (see picture below).



   (b) $\int_{-1}^{1} |x^3 - x|dx$

   $x^3 - x$ is negative on $[-1, 0]$ and positive on $[0, 1]$ so that

$$|x^3 - x| = \begin{cases} x^3 - x & x \in [-1, 0] \\ x - x^3 & x \in [0, 1] \end{cases}$$

   We have

$$\begin{aligned} \int_{-1}^{1} |x^3 - x|dx &= \int_{-1}^{0} (x^3 - x)dx + \int_{0}^{1} (x - x^3)dx \\ &= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]\Bigg|_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]\Bigg|_{0}^{1} \\ & 0 - \left(-\frac{1}{4}\right) + \left(\frac{1}{4} - 0\right) = \frac{1}{2}. \end{aligned}$$

(c) $\int_{-\pi}^{\pi} (\sin x - x)dx$

We have

$$\int_{-\pi}^{\pi} (\sin x - x)dx = -\cos x - \frac{x^2}{2}\Big|_{-\pi}^{\pi} = \left[-\cos(-\pi) - \frac{(-\pi)^2}{2}\right] - \left[-\cos(\pi) - \frac{\pi^2}{2}\right] = 0$$

(Once again, $\sin x - x$ is odd and $[-\pi, \pi]$ is symmetric about the y-axis.)

(d) $\int_{-\pi}^{\pi} |\sin x| dx$

$\sin x$ is negative on $[-\pi, 0]$ and positive on $[0, \pi]$ so that

$$|\sin x| = \begin{cases} -\sin x & x \in [-\pi, 0] \\ \sin x & x \in [0, \pi] \end{cases}$$

We have

$$\begin{aligned} \int_{-\pi}^{\pi} |\sin x| dx &= -\int_{-\pi}^{0} \sin x dx + \int_{0}^{\pi} \sin x dx \\ &= \cos x\Big|_{-\pi}^{0} - \cos x\Big|_{0}^{\pi} = (1 - (-1)) - (-1 - 1) = 4. \end{aligned}$$

2. Find the following derivatives

For these, you should know that

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(b(x))a'(x).$$

(a) $\frac{d}{dx}\int_{x^2}^{x^3} \frac{1}{1+x^2}dx$

$$= \frac{3x^2}{1+(x^3)^2} - \frac{2x}{1+(x^2)^2}.$$

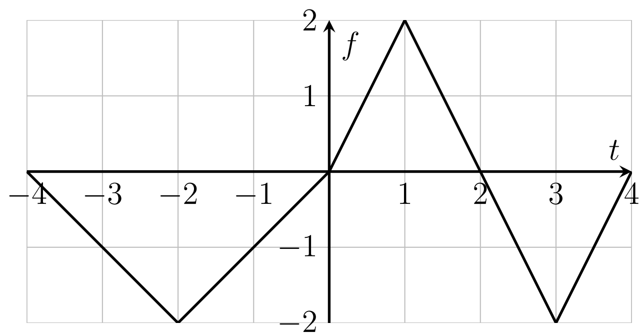
[You can also do this by actually computing the definite integral

$$\begin{aligned} \frac{d}{dx}\int_{x^2}^{x^3} \frac{1}{1+x^2}dx &= \frac{d}{dx}\left(\arctan x\Big|_{x^2}^{x^3}\right) \\ &= \frac{d}{dx}\left(\arctan x^3 - \arctan x^2\right) = \frac{3x^2}{1+(x^3)^2} - \frac{2x}{1+(x^2)^2}.] \end{aligned}$$

(b) $\frac{d}{dx}\int_{x^2}^{x^3} e^{-x^2}dx$

$$= e^{-(x^3)^2}3x^2 - e^{-(x^2)^2}2x.$$

3. Consider the function $f(t)$ whose graph is shown below, and let $g(x) = \int_0^x f(t)dt$.



(a) Find $g(-2)$, $g(0)$, $g(2)$, $g(3)$

$$g(-2) = 2,\ g(0) = 0,\ g(2) = 2,\ g(3) = 1.$$

(b) For what values of x is $g(x) = 0$?

$$g(0) = 0,\ g(4) = 0.$$

(c) On what intervals is g increasing/decreasing?
g is increasing where $g' = f$ is positive, on $(0, 2)$, and g is decreasing where $g' = f$ is negative, on $(-4, 0) \cup (2, 4)$.

(d) On what intervals is g concave up/concave down?
g is concave up where $g'' = f'$ is positive, on $(-2, 0) \cup (0, 1) \cup (3, 4)$, and g is concave down where $g'' = f'$ is negative, on $(-4, -2) \cup (1, 3)$.