

1. Find the following limits if they exist. MAKE SURE L'HOSPITAL'S RULE APPLIES BEFORE USING IT.

(a)

$$\lim_{x \rightarrow \infty} x^{1/x}, \quad \lim_{x \rightarrow 0^+} x^{1/x}$$

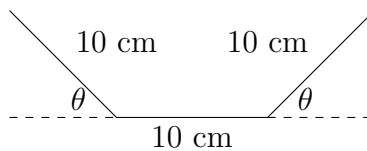
(b)

$$\lim_{x \rightarrow \infty} x \tan(1/x), \quad \lim_{x \rightarrow 2/\pi^+} x \tan(1/x)$$

(c)

$$\lim_{x \rightarrow 0^+} (\sin x)^{\sin x}, \quad \lim_{x \rightarrow \pi/2} \sin x^{\sin x}$$

2. A long sheet of metal of width 30 cm will be bent into a gutter as shown. Find  $\theta$  such that the gutter will have maximum capacity.



3. (Bonus) Find the minimum length of the line segment from the  $y$ -axis to the  $x$ -axis going through the point  $(a, b)$  in the first quadrant ( $a, b > 0$ ). [Answer:  $(a^{2/3} + b^{2/3})^{3/2}$ .]