Math 1300-018 Quiz 8 Name:________________________

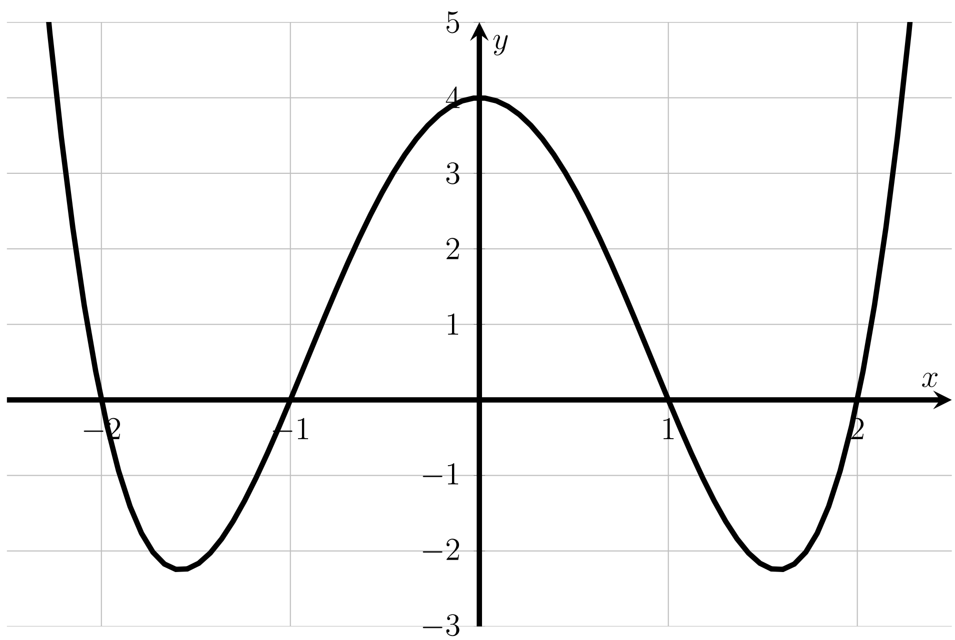
1. Consider the function $f(x) = x^4 - 5x^2 + 4$

   (a) Find the zeros of f.
   We have
   $$f(x) = (x^2 - 4)(x^2 - 1) = (x - 2)(x + 2)(x - 1)(x + 1)$$
   so that $f(x) = 0$ if $x = \pm 2, \pm 1$.

   (b) On what intervals is f increasing/decreasing? Find and classify all local extrema of f.
   We have
   $$f'(x) = 4x^3 - 10 = 0 \Longrightarrow x = 0, \pm\sqrt{5/2}.$$
   On $(-\infty, -\sqrt{5/2})$ f is decreasing, on $(-\sqrt{5/2}, 0)$ f is increasing, on $(0, \sqrt{5/2})$ f is decreasing, and on $(\sqrt{5/2}, \infty)$ f is increasing. From this we see that f has a local minimum at $x = 0$ of $f(0) = 4$, and that f has two local maxima at $\pm\sqrt{5/2}$ of $f(\pm\sqrt{5/2}) = -9/14$.

   (c) On what intervals is f concave up/concave down? Find any inflection points on the graph of $y = f(x)$.
   We have
   $$f''(x) = 12x^2 - 10 = 0 \Longrightarrow x = \pm\sqrt{5/6}.$$
   On $(-\infty, -\sqrt{5/6})$ f is CCU, on $(-\sqrt{5/6}, \sqrt{5/6})$ f is CCD, and on $(\sqrt{5/6}, \infty)$ f is CCU. There are two inflection points, $(\pm\sqrt{5/6}, 19/36)$.

   (d) Sketch a graph of $y = f(x)$ on the axes below using the above information. Include coordinates of all local extrema and inflection points.



2. Find the global extrema of $g(x) = (x^2 - 2)e^{x^2-2}$ on the interval $[-2, 3]$.

We have

$$\begin{aligned} g'(x) &= (x^2 - 2)e^{x^2-2}(2x) + (2x)e^{x^2-2} \\ &= (x^2 - 1)(2x)e^{x^2-2} \\ &= (x - 1)(x + 1)(2x)e^{x^2-2} \end{aligned}$$

so that the critical points of g are $0, 2$ and 3 (and the endpoints, depending on convention). We test the values of g at the critical points (and the endpoints) to find

$$g(-2) = 0, g(3) = 7e^7, g(-1) = -1/e, g(0) = -2e^{-2}, g(1) = -1/e.$$

The largest of these values is e (so the global maximum occurs twice) and the smallest of the values is $-1/e$ (so that the global minimum occurs twice). To see that $-1/e < -2/e^2$ without a calculator, note that

$$-1/e < -2/e^2 \iff -e = -2.718281828... < -2.$$