Math 1300-018 Quiz 6 Name:________________________

1. (a) $\frac{d}{dt}\arctan(\arcsin(\sqrt{t})) =$

$$\frac{1}{1+(\arcsin(\sqrt{t}))^2}\frac{1}{\sqrt{1-t}}\frac{1}{2\sqrt{t}}.$$

(b) $\frac{d^2}{dx^2}\arccos e^x =$

We have

$$\begin{aligned}\frac{d}{dx}\arccos e^x &= \frac{-e^x}{\sqrt{1-e^{2x}}}\\ \frac{d^2}{dx^2}\arccos e^x &= \frac{d}{dx}\frac{-e^x}{\sqrt{1-e^{2x}}} = \frac{-e^x\sqrt{1-e^{2x}}+e^x\left(\frac{-2e^{2x}}{2\sqrt{1-e^{2x}}}\right)}{1-e^{2x}}\\ &= \frac{-e^x}{(1-e^{2x})^{3/2}}.\end{aligned}$$

2. Find the equation of the tangent line to the curve

$$e^y\cos x = \sin(xy) - e$$

through the point $(\pi, 1)$.

Differentiating with respect to $x$ (with $y = y(x)$ a function of $x$), we get

$$-e^y\sin x + y'e^y\cos x = \cos(xy)(xy'+y).$$

Evaluating at $(x,y) = (\pi,1)$ gives

$$-ey' = -(\pi y'+1)$$

so that $y' = \frac{1}{e-\pi}$ when $(x,y) = (\pi, e)$. Or you could solve for $y'$, then evaluate at $(\pi,1)$:

$$y' = \frac{y\cos(xy)+e^y\sin x}{e^y\cos x - x\cos(xy)},\ y'\Big|_{(\pi,1)} = \frac{-1}{-e+\pi}.$$

The equation of the tangent line is therefore

$$y-1 = \frac{1}{e-\pi}(x-\pi).$$

3. Consider the function

$$f(x) = 4x^3 + 2x - 1.$$

(a) Show that $f$ is invertible.

The function is invertible because it is always increasing, $f'(x) = 12x^2 + 2 > 0$.

(b) Find $\left(f^{-1}\right)'(-1)$.

In general, $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$. So we have to find $x_0 = f^{-1}(-1)$ and $f(x_0)$ to evaluate $(f^{-1})'(-1)$. By inspection, $f(0) = -1$ so that $f^{-1}(-1) = 0$. Next, $f'(0) = 2$. Hence

$$(f^{-1})'(-1) = \frac{1}{f'(f^{-1}(-1))} = \frac{1}{2}.$$