Math 1300-018 Quiz 4 Name:_________________________

1. Find the derivatives of the following functions (using the definition of the derivative as a limit).

   (a) $f(x) = \sqrt{2x-1}$

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{2(x+h)-1} - \sqrt{2x-1})(\sqrt{2(x+h)-1} + \sqrt{2x-1})}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})} \\
&= \lim_{h\to 0} \frac{(2(x+h)-1) - (2x-1)}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})} \\
&= \lim_{h\to 0} \frac{2h}{h(\sqrt{2(x+h)-1} + \sqrt{2x-1})} \\
&= \lim_{h\to 0} \frac{2}{\sqrt{2(x+h)-1} + \sqrt{2x-1}} \\
&= \frac{2}{\sqrt{2(x+0)-1} + \sqrt{2x-1}} = \frac{1}{\sqrt{2x-1}}.
\end{aligned}$$

   (b) $g(x) = \dfrac{1}{3-2x}$

$$\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{\frac{1}{3-2(x+h)} - \frac{1}{3-2x}}{h} \\
&= \lim_{h\to 0} \frac{(3-2x) - (3-2(x+h))}{h(3-2x)(3-2(x+h))} \\
&= \lim_{h\to 0} \frac{2h}{h(3-2x)(3-2(x+h))} \\
&= \lim_{h\to 0} \frac{2}{(3-2x)(3-2(x+h))} \\
&= \frac{2}{(3-2x)(3-2(x+0))} = \frac{2}{(3-2x)^2}.
\end{aligned}$$

2. Consider the function

$$f(t) = t^3 - t^2 - t + 1.$$

(a) The derivative of f is given by $f'(t) = 3t^2 - 2t - 1$. Using this, find the largest open intervals on which f is increasing and decreasing. At what values of t does f have local extrema?

The derivative $f'(t)$ factors as $(3t+1)(t-1)$ with roots $-1/3, 1$. The derivative is positive on $(-\infty, -1/3) \cup (1, \infty)$ and negative on $(-1/3, 1)$. There is a local maximum of $f(-1/3) = 11/9$ at $t = -1/3$ and a local minimum of $f(1) = 0$ at $t = 1$.

(b) The second derivative of f is given by $f''(t) = 6t - 2$. Using this, find the largest open intervals on which f is concave up and concave down. Find any inflection points on the graph of f.

The second derivative has only root, at $t = 1/3$. The second derivative is positive on $(1/3, \infty)$ and negative on $(-\infty, 1/3)$, so that f is concave up on $(1/3, \infty)$ and concave down on $(-\infty, 1/3)$. There is an inflection point of $(1/3, f(1/3)) = (1/3, 16/27)$ where the concavity changes.