Math 1300-018 Quiz 3 Name:_____

This is a take-home quiz, due Monday, September 14th in class. Work on your own and SHOW YOUR WORK AND EXPLAIN YOUR REASONING. Use additional paper as needed.

1. Consider the rational function

$$r(x) = \frac{3x^2 - 6x - 24}{x^4 - x^3 - 6x^2}.$$

   (a) Where is $r(x)$ discontinuous?

   (b) For each value $x = a$ from (a), find $\lim_{x \to a^+} r(x)$ and $\lim_{x \to a^-} r(x)$ (including limits of $\pm\infty$).

   (c) Find $\lim_{x \to \infty} r(x)$.

   (d) Sketch a graph of $r(x)$ including zeros and horizontal and vertical asymptotes.

2. Consider the following function (where $a, b \in \mathbb{R}$ are constants)

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < -2 \\ ax^2 + bx + 1 & \text{if } -2 \leq x < 3 \\ ax + b & \text{if } x \geq 3 \end{cases}$$

   (a) Determine $\lim_{x \to -2^-} f(x)$, $\lim_{x \to -2^+} f(x)$, $\lim_{x \to 3^-} f(x)$, and $\lim_{x \to 3^+} f(x)$ (your answers may include the constants a and b).

   (b) Find values of a and b that will make f continuous on $(-\infty, \infty)$. [Write down two equations in the two unknowns a and b, one to describe continuity at $x = -2$ and another to describe continuity at $x = 3$.]

3. Use the intermediate value theorem to show that $x^2 = e^x$ has a solution (do not try to solve for x exactly).

4. Find

$$\lim_{x \to \infty} \sqrt{x^2 + ax} - \sqrt{x^2 + bx}.$$

   [Hint: Get rid of the square roots using $x^2 - y^2 = (x - y)(x + y)$ then multiply and divide by an appropriate power of x. Your answer will be a finite number depending on the constants a and b.]