Math 1300-018 Quiz 2, September 8, 2015 Name:_____

1. Consider the function $f(x) = x^3$.

   (a) Find an expression for the slope $m(x)$ of the line through the points $(1, 1)$ and $(x, f(x))$ (i.e. the slope of the secant line through the two points).
   The slope of the line through the points $(1, 1)$ and (x, x^3) is

   $$m(x) = \frac{x^3 - 1}{x - 1}.$$

   (b) The function $m(x)$ from part (a) is not defined at $x = 1$, but $\lim_{x\to 1} m(x)$ exists. (This is the slope of the tangent line to the curve $y = f(x)$ at $x = 1$.) Find $\lim_{x\to 1} m(x)$ and give an equation for the tangent line to $y = f(x)$ through the point $(1, 1)$.
   We have

   $$\lim_{x\to 1} m(x) = \lim_{x\to 1} \frac{x^3 - 1}{x - 1} = \lim_{x\to 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1} = \lim_{x\to 1} x^2 + x + 1 = 3$$

   so that the equation of the tangent line is $y - 1 = 3(x - 1)$.

2. Consider the function

$$g(x) = \begin{cases} x+1 & x < 1 \\ 1 & x = 1 \\ x^2 - 4x + 5 & x > 1 \end{cases}.$$

(a) What is $\lim_{x \to 1^+} g(x)$?

We have

$$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} x^2 - 4x + 5 = 2.$$

(b) What is $\lim_{x \to 1^-} g(x)$?

$$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} x + 1 = 2.$$

(c) Does $\lim_{x \to 1} g(x)$ exist, and if so, what is its value?

$\lim_{x \to 1} g(x) = 2$ because the left and right limits at $x = 1$ exist and are equal to 2.

(d) Sketch a graph of $y = g(x)$ on the axes below.