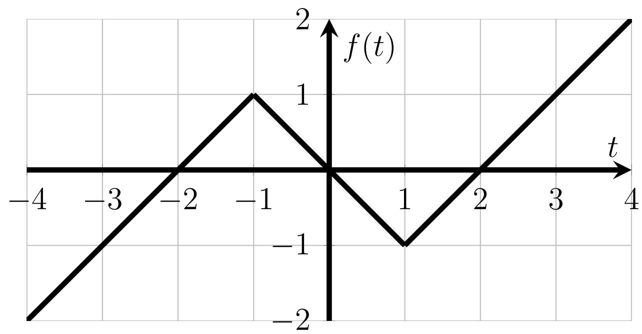
Math 1300-018 Quiz 11 Name:________________________

1. Below is the graph of of $f(t), t \in \mathbb{R}$. Let $F(x) = \int_0^x f(t)dt$.



(a) Without finding F explicitly, answer the following:

i. What is $F(3)$? $F'(3)$? $F''(3)$?

$$F(3) = -1/2,\ F'(3) = f(3) = 1,\ F''(3) = f'(3) = 1$$

ii. What is $F(-3)$? $F'(-3)$? $F''(-3)$?

$$F(-3) = -1/2,\ F'(-3) = f(-3) = -1,\ F''(-3) = f'(-3) = 1$$

iii. For what values of x is $F(x) = 0$?

$$x = 0, 2+\sqrt{2}, -2-\sqrt{2}$$

iv. On what intervals is F increasing/decreasing?
F is increasing where $F' = f$ is positive, $(-2,0) \cup (2,\infty)$. F is decreasing where $F' = f$ is negative, $(-\infty,-2) \cup (0,2)$.

v. Find and classify all local extrema of F.
F has a local maximum of 0 at $x = 0$ and local minima of -1 at $x = \pm 2$.

vi. On what intervals is F concanve up/down?
F is concave up where $F'' = f'$ is positive, $(-\infty,-1) \cup (1,\infty)$. F is concave down where $F'' = f'$ is negative, $(-1,1)$.

vii. Find any inflection points of F.
F has inflection points where $F'' = f'$ doesn't exst, at $x = \pm 1$. These inflection points are $(\pm 1, -1/2)$.

(b) Find an explicit formula for $f(t)$.

$$f(t) = \begin{cases} t+2 & t \le -1 \\ -t & -1 \le t \le 1 \\ t-2 & t \ge 1 \end{cases}.$$

(c) Find an explicit forumla for $F(x)$ and sketch a graph of F. [You may want to use the "recovery" interpretation of the fundamental theorem of calculus: if g' is continuous on an interval containing a and x then $g(x) = g(a) + \int_a^x g'(t)dt$.]

On $[-1, 1]$ we have

$$F(x) = \int_0^x f(t)dt = \int_0^x -tdt = -\frac{x^2}{2}.$$

On $[1, \infty)$ we have

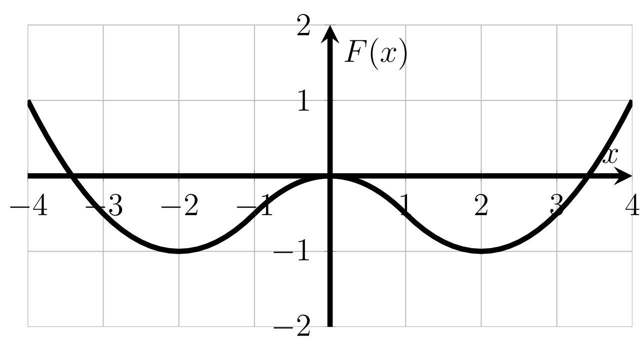
$$F(x) = \int_0^1 f(t)dt + \int_1^x f(t)dt = -1/2 + \int_1^x (t-2)dt = -\frac{1}{2} + \left(\frac{t^2}{2} - 2t\right)\Big|_1^x = \frac{x^2}{2} - 2x + 1.$$

On $(-\infty, -1]$ we have

$$F(x) = \int_0^{-1} f(t)dt + \int_{-1}^x f(t)dt = -1/2 - \int_x^{-1} (t+2)dt = -\frac{1}{2} - \left(\frac{t^2}{2} + 2t\right)\Big|_x^{-1} = \frac{x^2}{2} + 2x + 1.$$

So altogether we have

$$F(x) = \begin{cases} \frac{x^2}{2} + 2x + 1 & (-\infty, -1] \\ -x^2/2 & -1 \le x \le 1 \\ \frac{x^2}{2} - 2x + 1 & [1, \infty) \end{cases}.$$



2. Compute the following indefinite and definite integrals.

(a) $\int (3^x + x^3)dx, \ \int_0^1 (3^x + x^3)dx$

$$\int (3^x + x^3)dx = \frac{3^x}{\ln 3} + \frac{x^4}{4} + C, \ \int_0^1 (3^x + x^3)dx = \frac{3^x}{\ln 3} + \frac{x^4}{4}\Big|_0^1 = \frac{2}{\ln 3} + \frac{1}{4}$$

(b) $\int \frac{x}{x+1}dx, \ \int_0^1 \frac{x}{x+1}dx$

$$\int \frac{x}{x+1}dx = \int \left(1 - \frac{1}{x+1}\right) dx = x - \ln|x+1| + C, \ \int_0^1 \frac{x}{x+1}dx = 1 - \ln 2$$

(c) $\int 2xe^{x^2}dx, \ \int_{-1}^1 2xe^{x^2}dx$

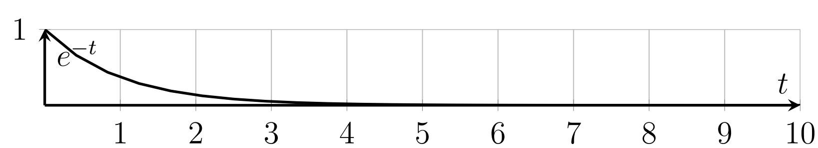
$$\int 2xe^{x^2}dx = e^{x^2} + C, \ \int_{-1}^1 2xe^{x^2}dx = 0$$

3. Find $F(x) = \int_0^x e^{-t}dt$ and $A = \lim_{x\to\infty} F(x)$. What area does A represent (draw a picture)?

$$F(x) = \int_0^x e^{-t}dt = -e^{-t}\Big|_0^x = 1 - e^{-x}$$

$$A = \lim_{x\to\infty} 1 - e^{-x} = 1 - 0 = 1$$

A is the *finite* area of the infinite stip between the x-axis and the graph of e^{-t} over the interval $[0, \infty)$. As you can see, e^{-t} gets very small, very quickly - quickly enough that A is finite.



4. What is wrong with the following computation?

$$\int_{-1}^3 \frac{dx}{x^2} = -\frac{1}{x}\Big|_{-1}^3 = -\frac{1}{3} - \left(-\frac{1}{-1}\right) = -\frac{4}{3}$$

The function $\frac{1}{x^2}$ is not integrable near $x = 0$ where it has a vertical asymptote, so the fundamental theorem of calculus does not apply.