7.3

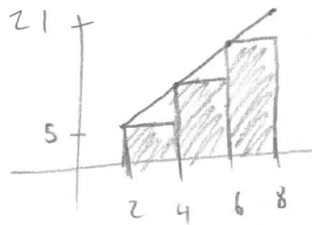
(3) $f(x) = 2x+5,\ x_1 = 0,\ x_2 = 2,\ x_3 = 4,\ x_4 = 6,\ \Delta x = 2$

$$\sum_{i=1}^{4} f(x_i)\,\Delta x = f(0)\cdot 2 + f(2)\cdot 2 + f(4)\cdot 2 + f(6)\cdot 2$$

$$= 2(5 + 9 + 13 + 17) = 88$$

$$\sum_{i=1}^{4} f(x_i)\,\Delta x \approx \int_0^8 (2x+5)\,dx$$

left endpoints



(7) $f(x) = 4 - x^2$ on $[-2, 2]$

left endpts $x = -2, -1, 0, 1$ $\quad 4-(-2)^2 + 4-(-1)^2 + 4-(0)^2 + 4-(1)^2 = 10$

right endpts $x = -1, 0, 1, 2$ $\quad 4-(-1)^2 + 4-(0)^2 + 4-(1)^2 + 4-(2)^2 = 10$

average $\quad \dfrac{10+10}{2} = 10$

midpts $x = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ $\quad 4-\left(-\frac{3}{2}\right)^2 + 4-\left(-\frac{1}{2}\right)^2 + 4-\left(\frac{1}{2}\right)^2 + 4-\left(\frac{3}{2}\right)^2$

$= 11$

actual $\displaystyle\int_{-2}^{2} (4-x^2)\,dx = 4x - \frac{x^3}{3}\Big|_{-2}^{2} = \left(8 - \frac{8}{3}\right)\cdot 2 = 32/3 = 10.\overline{6}$

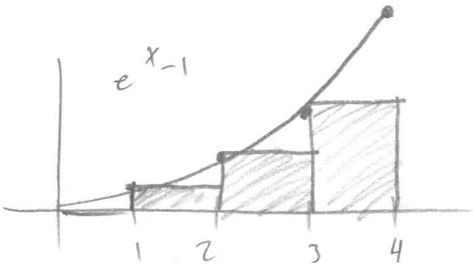
(9) $f(x) = e^x - 1$ on $[0,4]$

left $(e^0-1) + (e^1-1) + (e^2-1) + (e^3-1) \approx$

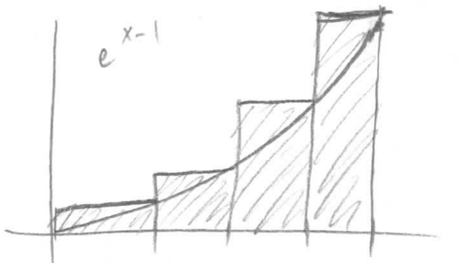
right $(e^1-1) + (e^2-1) + (e^3-1) + (e^4-1) \approx$

avg $\frac{\text{right}+\text{left}}{2} \approx$

mid $(e^{1/2}-1) + (e^{3/2}-1) + (e^{5/2}-1) + (e^{7/2}-1) \approx$



left



right

mid

actual $\int_0^4 (e^x - 1)\,dx = e^x - x\Big|_0^4 = (e^4 - 4) - (e^0 - 0) = e^4 - 5 \approx$

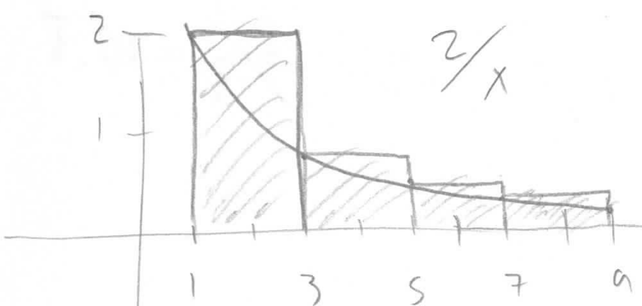
(11) $f(x) = \frac{2}{x}$ on $[1,9]$

left $\frac{2}{1}\cdot 2 + \frac{2}{3}\cdot 2 + \frac{2}{5}\cdot 2 + \frac{2}{7}\cdot 2 =$

right $\frac{2}{3}\cdot 2 + \frac{2}{5}\cdot 2 + \frac{2}{7}\cdot 2 + \frac{2}{9}\cdot 2 =$

avg

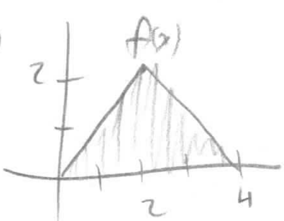
mid $\frac{2}{2}\cdot 2 + \frac{2}{4}\cdot 2 + \frac{2}{6}\cdot 2 + \frac{2}{8}\cdot 2 =$



left

etc.

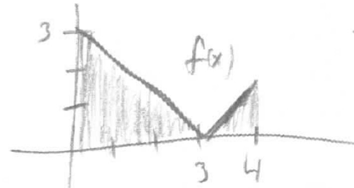
(15) (a)



$\frac{1}{2} \cdot 4 \cdot 2 = 4$

$= \int_0^4 f(x)\,dx$

(b)



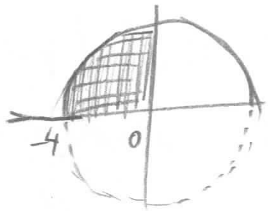
$\frac{1}{2} \cdot 3 \cdot 3 + \frac{1}{2} = 5$

$= \int_0^4 f(x)\,dx$

(17) $\int_{-4}^{0} \sqrt{16-x^2}\,dx = \frac{\pi(4)^2}{4} = 4\pi$

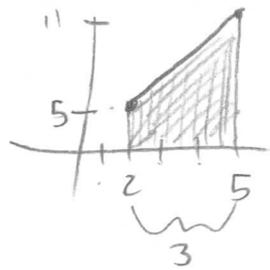
$y = \sqrt{16-x^2}$

$x^2+y^2 = 16 = 4^2$



$\frac{\pi r^2}{4}$, $r=4$

(19) $\int_2^5 (1+2x)\,dx$



$\frac{1}{2} \cdot (5+11) \cdot 3 = 24$

(25)

left $3(34+57+115+264) = 1410$

right $3(57+115+264+697) = 3399$

avg 2404.5

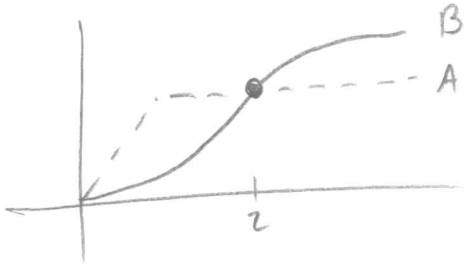
(27) Cattle $0 + 160 + 270 + 50 + 40 + 25 + 20 + 10$
$= 575{,}000$

Pigs $0 + 60 + 70 + 5 + 0 + 10 + 0 + 0 = 145{,}000$

(using left endpoints above)

Aug. 5th

(35)

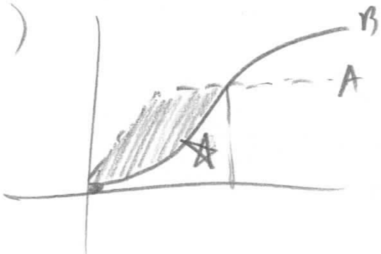


(a) 9 ft.

(b) 2 seconds

(First A goes faster, at 2 seconds B matches speed, then B starts closing the gap)

(c)



The shaded area is the furthest B gets ahead $= 9 - ☆ \approx 4.75$

$☆ \approx .25\frac{1}{2} + 1\cdot\frac{1}{2} + 3\frac{1}{2} + 4.25\frac{1}{2} = 4.25$

7.4 8/5/14

① $\int_{-2}^{4}(-3)\,dp = -3p\Big|_{-2}^{4} = -12-6 = -18$

③ $\int_{-1}^{2}(5t-3)\,dt = \frac{5t^2}{2} - 3t\Big|_{-1}^{2} = (10-6)-(\frac{5}{2}+3) = \frac{-3}{2}$

⑤ $\int_{0}^{2}(5x^2-4x+2)\,dx = \frac{5x^3}{3} - \frac{4x^2}{2} + 2x\Big|_{0}^{2} = \left(\frac{40}{3} - \frac{8}{2} + 4\right) - 0$

$= \frac{40}{3}$

⑦ $\int_{0}^{2} 3\sqrt{4u+1}\,du = \int_{1}^{9}\frac{3}{4}\sqrt{u}\,du = \frac{2}{3}\frac{3}{4}u^{3/2}\Big|_{1}^{9}$

$x = 4u+1 \quad x=0, u=1$

$dx = 4du \quad x=2, u=9$

$= \frac{9^{3/2}}{2} - \frac{1}{2} = 13$

⑨ $\int_{0}^{4} 2(t^{1/2}-t)\,dt = \frac{2}{3}2t^{3/2} - 2\frac{t^2}{2}\Big|_{0}^{4} = \left(\frac{4}{3}4^{3/2} - 16\right) - (0)$

$= \frac{32}{3} - 16 = \frac{-16}{3}$

⑪ $\int_{1}^{4}(5y\sqrt{y} + 3\sqrt{y})\,dy = \int_{1}^{4}(5y^{3/2} + 3y^{1/2})\,dy = 5\frac{2}{5}y^{5/2} + 3\frac{2}{3}y^{3/2}\Big|_{1}^{4}$

$= \left[2(4^{5/2}) + 2(4^{3/2})\right] - \left[2+2\right] = 64+16-4 = 76$

⑬ $\int_{4}^{6}\frac{2}{(2x-7)^2}\,dx = \int_{1}^{5}u^{-2}\,du = -u^{-1}\Big|_{1}^{5} = -\frac{1}{5} - (-1)$

$= \frac{4}{5}$

$u = 2x-7 \quad x=6, u=5$

$du = 2dx \quad x=4, u=1$

(15) $\int_1^5 (6n^{-2} - n^{-3})\, dn = \frac{6n^{-1}}{-1} - \frac{n^{-2}}{-2}\Big|_1^5 = \left(\frac{-6}{5} + \frac{1}{50}\right) - \left(-6 + \frac{1}{2}\right)$

$= \frac{-59}{50} + \frac{300}{50} - \frac{25}{50} = \frac{216}{50} = \frac{108}{25}$

(17) $\int_{-3}^{-2} \left(2e^{-y/10} + 3/y\right) dy = -20e^{-y/10} + 3\ln|y|\Big|_{-3}^{-2} = \left(-20e^{-1/5} + 3\ln 2\right) - \left(-20e^{-3/10} + 3\ln 3\right)$

$= 20(e^{-3/10} - e^{-1/5}) + 3\ln 2/3 \approx$

(19) $\int_1^2 \left(e^{4u} - \frac{1}{(u+1)^2}\right) du = \frac{1}{4}e^{4u} + \frac{1}{u+1}\Big|_1^2 = \left(\frac{e^8}{4} + \frac{1}{3}\right) - \left(\frac{e^4}{4} + \frac{1}{2}\right)$

(21) $\int_{-1}^{0} y(2y^2-3)^5 dy = \int_{-1}^{-3} \frac{1}{4} u^5 du = \frac{u^6}{24}\Big|_{-1}^{-3} = \frac{3^6}{24} - \frac{1}{24}$

$= \frac{728}{24} = \frac{91}{3}$

$u = 2y^2 - 3$

$du = 4y\, dy$

(23) $\int_1^{64} \frac{\sqrt{z} - 2}{\sqrt[3]{z}} dz = \int_1^{64} \left(z^{1/6} - 2z^{-1/3}\right) dz = \frac{6}{7} z^{7/6} - 3z^{2/3}\Big|_1^{64}$

$= \left(\frac{6}{7} \cdot 128 - 3 \cdot 16\right) - \left(\frac{6}{7} - 3\right) \approx 63.857$

(25) $\int_1^2 \frac{\ln x}{x} dx = \int_0^{\ln 2} u\, du = \frac{u^2}{2}\Big|_0^{\ln 2} = \frac{(\ln 2)^2}{2}$

$u = \ln x \quad x = 1, u = 0$

$du = \frac{1}{x} dx \quad x = 2, u = \ln 2$

27 $\int_0^8 x^{1/3}\sqrt{x^{4/3}+9}\,dx = \int_9^{25} \frac{3}{4}\sqrt{u}\,du = \frac{u^{3/2}}{2}\Big|_9^{25}$

$u = x^{4/3}+9$ $\quad x=0,\ u=9$

$du = \frac{4}{3}x^{1/3}dx$ $\quad x=8,\ u=25$

$= \frac{125}{2} - \frac{27}{2} = \frac{98}{2}$

$= 49$

29 $\int_0^1 \frac{e^{2t}}{(3+e^{2t})^2}\,dt = \int_4^{3+e^2} \frac{1}{2}u^{-2}\,du =$

$-\frac{1}{2u}\Big|_4^{3+e^2} = \frac{1}{8} - \frac{1}{2(3+e^2)}$

$u = 3+e^{2t}$ $\quad t=1,\ u=3+e^2$

$du = 2e^{2t}dt$ $\quad t=0,\ u=4$