

9.3

July 30th

$$(21) \quad z = -3xy + x^3 - y^3 + \frac{1}{8}$$

$$\begin{cases} z_x = -3y + 3x^2 = 0 \Rightarrow y = x^2 \\ z_y = -3x - 3y^2 = 0 \Rightarrow x = -y^2 \end{cases} \Rightarrow \begin{cases} x = -x^4 \\ x^4 + x = 0 \\ x(x^3 + 1) = 0 \\ x = 0, -1 \end{cases}$$

If $x=0, y=0$ crit. pts. $(0,0)$
 If $x=-1, y=1$ $(-1,1)$

$$z_{xx} = 6x$$

$$z_{yy} = -6y$$

$$z_{xy} = -3 = z_{yx}$$

$$D_z(x,y) = -36xy - 9$$

$$D_z(0,0) = -9 < 0 \quad \text{saddle at } (0,0)$$

$$D_z(-1,1) = 25 > 0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{max at } (-1,1)$$

$$z_{xx}(-1,1) = -6 < 0 \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{of } z(-1,1) = \frac{9}{8}$$

(concave down)

Corresponds to graph (A)

22 $z = \frac{3}{2}y - \frac{y^3}{2} - x^2y + \frac{1}{16}$ $\begin{cases} z_x = -2xy = 0 \\ z_y = \frac{3}{2} - \frac{3}{2}y^2 - x^2 = 0 \end{cases}$

$\Rightarrow x=0$ or $y=0$ (from $z_x=0$)

$(0, 1)$	$(\frac{\sqrt{3}}{2}, 0)$
$(0, -1)$	$(-\frac{\sqrt{3}}{2}, 0)$

crit.
pts.

$x=0$ $\frac{3}{2} - \frac{3}{2}y^2 = 0$ $y = \pm 1$ $y=0$ $x = \pm \sqrt{\frac{3}{2}}$

$z_{xx} = -2y$

$D_z(x, y) = 6y^2 - 4x^2$

$z_{yy} = -3y$

$D_z(0, 1) = 6, z_{xx}(0, 1) = -2 < 0$

$z_{yx} = z_{xy} = -2x$

• max of $17/16$ at $(0, 1)$

$D_z(0, -1) = 6, z_{xx}(0, -1) = 2 > 0$

• min of $-15/16$ at $(0, -1)$

graph

$D_z(\frac{\sqrt{3}}{2}, 0) < 0$ saddle at $(\frac{\sqrt{3}}{2}, 0)$

$D_z(-\frac{\sqrt{3}}{2}, 0) < 0$ saddle at $(-\frac{\sqrt{3}}{2}, 0)$

ⓓ

$$\textcircled{23} \quad z = y^4 - 2y^2 + x^2 - \frac{17}{16} \quad \left. \begin{array}{l} z_x = 2x \\ z_y = 4y^3 - 4y \end{array} \right\} \Rightarrow \begin{array}{l} x=0 \\ y=0, \pm 1 \end{array}$$

crit. pts.

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$$

$$z_{xx} = 2, \quad z_{yy} = 12y^2 - 4, \quad z_{xy} = 0$$

$$D_z(x,y) = 8(3y^2 - 1)$$

• $D_z(0,0) = -8 < 0$ saddle at $(0,0)$

• $D_z(0,1) = 16 > 0, \quad z_{xx}(0,1) = 2 > 0$ min of $\frac{-33}{16}$ at $(0,1)$
(ccu)

• $D_z(0,-1) = 16 > 0, \quad z_{xx} = 2 > 0$ min of $\frac{-33}{16}$ at $(0,-1)$

graph $\textcircled{\beta}$

$$\textcircled{24} \quad z = -2x^3 - 3y^4 + 6xy^2 + \frac{1}{16} \quad \begin{cases} z_x = -6x^2 + 6y^2 = 0 \\ z_y = -12y^3 + 12xy = 0 \end{cases}$$

$$\Rightarrow \left. \begin{array}{l} x^2 = y^2, \quad x = \pm y \\ (z_x = 0) \end{array} \right\} \Rightarrow$$

$$\underline{x=y} \quad y^2 = y^3, \quad y = 0, 1$$

$$\Rightarrow \left. \begin{array}{l} xy = y^3 \\ (z_y = 0) \end{array} \right\} \Rightarrow$$

$$\underline{x=-y} \quad -y^2 = y^3, \quad y = 0, -1$$

crit. pts

$$\begin{pmatrix} 0, 0 \\ 1, 1 \\ 1, -1 \end{pmatrix}$$

$$z_{xx} = -12x, \quad z_{yy} = -36y^2 + 12x, \quad z_{xy} = 12y$$

$$D_z(x,y) = (-12x)(12x - 36y^2) - 144y^2$$

• $D_z(0,0) = 0$???

• $D_z(1,1) > 0, \quad z_{xx}(1,1) = -12$ max of $\frac{17}{16}$
(ccv)

• $D_z(1,-1) > 0, \quad z_{xx}(1,-1) = -12$ max of $\frac{17}{16}$

graph

\textcircled{C}

(25) $z = -x^4 + y^4 + 2x^2 - 2y^2 + \frac{1}{16}$ $\begin{cases} z_x = -4x^3 + 4x = 0 \\ z_y = 4y^3 - 4y = 0 \end{cases}$

$\Rightarrow x = x^3, x = 0, \pm 1$
 $y = y^3, y = 0, \pm 1$

<u>crit pts</u>	(0,0)	(1,0)	(-1,0)
	(0,1)	(1,1)	(-1,1)
	(0,-1)	(1,-1)	(-1,-1)

$z_{xx} = -12x^2 + 4$ $D_z(x,y) = -16(3x^2-1)(3y^2-1)$

$z_{yy} = 12y^2 - 4$

$z_{xy} = 0 = z_{yx}$

$D_z(0,0) < 0$ saddle

$D_z(0,1) > 0, z_{xx} > 0$ min

$D_z(0,-1) > 0, z_{xx} > 0$ min

$D_z(-1,0) > 0, z_{xx} < 0$ max

$D_z(1,1) < 0$
 $D_z(-1,-1) < 0$ } saddle

$D_z(1,0) > 0, z_{xx} < 0$ max

$D_z(1,1) < 0$ } saddle

$D_z(1,-1) < 0$ }

graph (E)

(26) $z = -y^4 + 4xy - 2x^2 + \frac{1}{16}$ $\begin{cases} z_x = 4y - 4x = 0 \\ z_y = -4y^3 + 4x = 0 \end{cases}$

$\Rightarrow x=y \Rightarrow y=y^3 \Rightarrow y=0, \pm 1$ | crit pts $\begin{pmatrix} 0,0 \\ 1,1 \\ -1,-1 \end{pmatrix}$

$z_{xx} = -4, z_{yy} = -12y^2, z_{xy} = 4 = z_{yx}$

$D_z(x,y) = 48y^2 - 16$

graph (F)

$D_z(0,0) = -16$ saddle

$D_z(1,1) = 32, z_{xx} < 0$ max

$D_z(-1,-1) = 32, z_{xx} < 0$ max

$$(34) P(x,y) = 1500 + 36x - \frac{3}{2}x^2 + 120y - 2y^2$$

$$\frac{\partial P}{\partial x} = 36 - 3x = 0 \Rightarrow x = 12$$

$$\frac{\partial P}{\partial y} = 120 - 4y = 0 \Rightarrow y = 30$$

$$P(12, 30) = 3516$$

$$(35) L(x,y) = \frac{3}{2}x^2 + y^2 - 2x - 2y - 2xy + 68$$

$$\frac{\partial L}{\partial x} = 3x - 2 - 2y = 0 \Rightarrow x = \frac{2}{3}(y+1)$$

$$\frac{\partial L}{\partial y} = 2y - 2 - 2x = 0 \Rightarrow y = x+1$$

$$\left. \begin{array}{l} \Rightarrow 3x = 2(y+1) \\ \Rightarrow 3x = 2(x+2) \\ x = 4 \\ y = 5 \end{array} \right\}$$

$$L(4,5) = 59$$