① $z = f(x,y) = 6x^2 - 4xy + 9y^2$

(a) $\dfrac{\partial z}{\partial x} = \lim\limits_{h\to 0}\left(\left[6(x+h)^2 - 4(x+h)y + 9y^2\right] - \left[6x^2 - 4xy + 9y^2\right]\right)\Big/h$

$= \lim\limits_{h\to 0}\left(12xh + 6h^2 - 4hy\right)\frac{1}{h}$

$= \lim\limits_{h\to 0} 12x + 6h - 4y = \boxed{12x - 4y}$

(b) $\dfrac{\partial z}{\partial y} = \lim\limits_{h\to 0}\left(\left[6x^2 - 4x(y+h) + 9(y+h)^2\right] - \left[6x^2 - 4xy + 9y^2\right]\right)\Big/h$

$= \lim\limits_{h\to 0}\left(-4xh + 18yh + 9h^2\right)\frac{1}{h} = \lim\limits_{h\to 0} -4x + 18y + 9h$

$= \boxed{-4x + 18y}$

(c) $f_x(2,3) = \lim\limits_{h\to 0}\dfrac{\left[6(2+h)^2 - 4(2+h)\cdot 3 + 9\cdot 3^2\right] - \left[6\cdot 2^2 - 4\cdot 2\cdot 3 + 9\cdot 3^2\right]}{h}$

$= \lim\limits_{h\to 0}\left(24h + 6h^2 - 12h\right)\frac{1}{h} = \lim\limits_{h\to 0} 24 + 6h - 12$

$= 12 \left(= 12\cdot 2 - 4\cdot 3 = \dfrac{\partial z}{\partial x}\Big|_{(2,3)} \text{ from (a)}\right)$

(d) $f_y(1,-2) = \lim\limits_{h\to 0}\frac{1}{h}\left(\left[6\cdot 1^2 - 4\cdot 1\cdot(-2+h) + 9(-2+h)^2\right] - \left[6\cdot 1^2 - 4\cdot 1\cdot(-2) + 9(-2)^2\right]\right)$

$= \lim\limits_{h\to 0}\left(-4h - 36h + 9h^2\right)\frac{1}{h} = \lim\limits_{h\to 0} -4 - 36 + 9h$

$= -40 \left(= -4\cdot 1 + 18(-2) = \dfrac{\partial z}{\partial y}\Big|_{(1,-2)} \text{ from (b)}\right)$

(3) $f(x,y) = -4xy + 6y^3 + 5$

$f_x(x,y) = -4y + 0 + 0$, $f_x(2,-1) = 4$

$= -4y$

$f_y(x,y) = -4x + 18y^2 + 0$, $f_y(-4,3) = 16 + 162 = 178$

$= -4x + 18y^2$

(5)

$f(x,y) = 5x^2y^3$

$f_x(x,y) = 5y^3(10x) = 50xy^3$, $f_x(2,-1) = -100$

$f_y(x,y) = 5x^2(3y^2) = 15x^2y^2$, $f_y(-4,3) = 2160$

(9) $f(x,y) = -6e^{4x-3y}$

$f_x(x,y) = -6 \cdot e^{4x-3y} \cdot (4)$ $f_x(2,-1) = -24e^{11}$

$= -24e^{4x-3y}$

$f_y(x,y) = -6e^{4x-3y}(-3)$ $f_y(-4,3) = 18e^{-25}$

$= 18e^{4x-3y}$

(11) $f(x,y) = \dfrac{x^2+y^3}{x^3-y^2}$

$f_x(x,y) = \dfrac{(x^3-y^2)(2x+0) - (x^2+y^3)(3x^2-0)}{(x^3-y^2)^2}$ $f_x(2,-1) = \dfrac{-8}{49}$

$\approx -.16$

$= \dfrac{2x^4 - 2xy^2 - 3x^4 - 3x^2y^3}{(x^3-y^2)^2} = \dfrac{-x^4 - 2xy^2 - 3x^2y^3}{(x^3-y^2)^2}$

(11) continued

$$f_y(x,y) = \frac{(x^3-y^2)(0+3y^2)-(x^2+y^3)(0-2y)}{(x^3-y^2)^2}$$

$$= (3x^3y^2-3y^4+2x^2y+2y^4)/(x^3-y^2)^2$$

$$= (-y^4+3x^3y^2+2x^2y)/(x^3-y^2)^2$$

$$f_y(-4,3) = \frac{-1713}{5329} \approx -.32$$

(13)

$$f(x,y) = \ln|1+5x^3y^2|$$

$$f_x(x,y) = \frac{1}{1+5x^3y^2}\left(5y^2(3x^2)\right) = \frac{15x^2y^2}{1+5x^3y^2}$$

$$f_y(x,y) = \frac{5x^3(2y)}{1+5x^3y^2} = \frac{10x^3y}{1+5x^3y^2}$$

$$f_x(2,-1) = \frac{60}{41} \approx 1.5, \quad f_y(-4,3) = \frac{1920}{2879} \approx .67$$

(15)

$$f(x,y) = xe^{x^2y}$$

$$f_x(x,y) = x(2xye^{x^2y}) + e^{x^2y}\cdot 1 = e^{x^2y}(1+2x^2y)$$

$$f_x(2,-1) = -7e^{-4}$$

$$f_y(x,y) = x(x^2e^{x^2y}) = x^3e^{x^2y}$$

$$f_y(-4,3) = -64e^{48}$$

(17) $f(x,y) = \sqrt{x^4+3xy+y^4+10}$

$f_x(x,y) = \frac{1}{2}(x^4+3xy+y^4+10)^{-1/2}(4x^3+3y)$

$f_y(x,y) = \frac{1}{2}(x^4+3xy+y^4+10)^{-1/2}(3x+4y^3)$

$f_x(2,-1) =$

$f_y(-4,3) =$

(19) $f(x,y) = \dfrac{3x^2y}{e^{xy}+2}$

$f_x(x,y) = \dfrac{(e^{xy}+2)(3y\cdot 2x)-(3x^2y)(ye^{xy})}{(e^{xy}+2)^2}$

$= e^{xy}(6xy-3x^2y^2)+12xy \Big/ (e^{xy}+2)^2$

$f_y(x,y) = \dfrac{(e^{xy}+2)(3x^2)-3x^2y(xe^{xy})}{(e^{xy}+2)^2}$

$= 3x^2e^{xy}(1-xy)+6x^2 \Big/ (e^{xy}+2)^2$

$f_x(2,-1) =$

$f_y(-4,3) =$

(21) $f(x,y) = 4x^2y^2 - 16x^2 + 4y$

$f_x = 8xy^2 - 32x$

$f_y = 8x^2y + 4$

$\begin{cases} f_{xx} = 8y^2 - 32 \\ f_{xy} = f_{yx} = 16xy \\ f_{yy} = 8x^2 \end{cases}$

(25) $r(x,y) = \dfrac{6y}{x+y}$

$r_x = \dfrac{-6y}{(x+y)^2}$

$r_{xy} = r_{yx} = \dfrac{-6(x+y)^2 + 6y \cdot 2(x+y)}{((x+y)^2)^2} = \dfrac{6(y-x)}{(x+y)^3}$

$r_y = \dfrac{6(x+y) - 6y}{(x+y)^2} = \dfrac{6x}{(x+y)^2}$

$r_{xx} = \dfrac{12y}{(x+y)^3}$ $\quad r_{yy} = \dfrac{-12x}{(x+y)^3}$

(29) $r = \ln|x+y|$

$r_x = \dfrac{1}{x+y}$ $\quad r_{xx} = \dfrac{-1}{(x+y)^2}$

$r_{xy} = r_{yx} = \dfrac{-1}{(x+y)^2}$

$r_y = \dfrac{1}{x+y}$ $\quad r_{yy} = \dfrac{-1}{(x+y)^2}$

(33) $f(x,y) = 6x^2 + 6y^2 + 6xy + 36x - 5$

$f_x = 12x + 6y + 36 = 0$

$f_y = 12y + 6x = 0$

$\begin{cases} 12x + 6y = -36 \\ 6x + 12y = 0 \end{cases}$

$-18y = -36$

$y = 2$

$x = -4$

$(x,y) = (-4, 2)$ is the only solution to $f_x = f_y = 0$

(35) $f(x,y) = 9xy - x^3 - y^3 6$

$f_x = 9y - 3x^2 = 0 \Rightarrow y = \frac{x^2}{3}$

$f_y = 9x - 3y^2 = 0 \Rightarrow x = \frac{y^2}{3}$

$y = \frac{y^4}{27}$

$\Rightarrow y = 0, 3$ (solutions of $y^4 - 27y = 0 = y(y^3 - 27) = y(y-3)(y^2+3y+9)$)

If $y = 0$, $x = 0$ $\left(\begin{matrix} 0 - 3x^2 = 0 \\ 9x - 0 = 0 \end{matrix}\right)$. If $y = 3$, $9 \cdot 3 - 3x^2 = 0 \Rightarrow x = \pm 3$

$9x - 3 \cdot 3^2 = 0 \Rightarrow x = 3$

So $(0,0)$ and $(3,3)$ are the solutions of the system

$f_x = f_y = 0$

(65) $F = m \cdot \frac{gR^2}{r^2} = \frac{1}{r^2} \cdot mgR^2$

$F_m = \frac{gR^2}{r^2}$ (how force changes with mass, lbs/slug)

$F_r = -\frac{mgR^2}{r^3}$ (how force changes with distance lbs/ft)

$F_m > 0$ (increase in mass increases force) $F_r < 0$ (increase in distance decreases force)