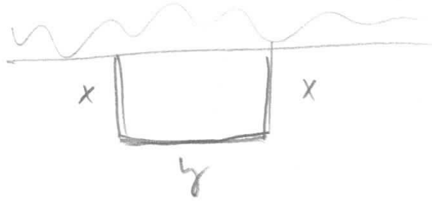
6.2 July 25

9




$2x + y = 1400$ (constraint)

(a) $y = 1400 - 2x$ (solving for y, eliminating a variable)

(b) $\text{Area} = xy = x(1400 - 2x) = A(x)$

(c) $A'(x) = 1400 - 4x = 0, \; x = \frac{1400}{4} = 350$ meters

(d) $A(350) = 350 \cdot 700 = 245{,}000 \text{ m}^2$

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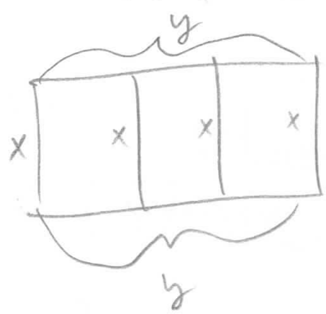



$4x + 2y = 3600$ (constraint)

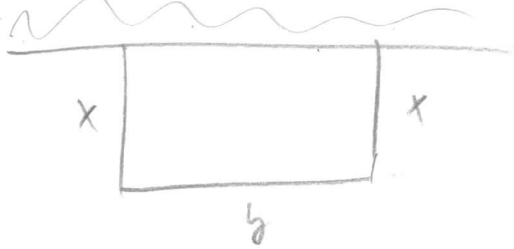
$y = \frac{3600 - 4x}{2}$ (eliminate a variable)

$\text{Area} = x \cdot y = x\left(\frac{3600 - 4x}{2}\right) = 1800x - 2x^2 = A(x)$

$A'(x) = 1800 - 4x = 0; \; x = \frac{1800}{4} = 450$ meters

$A(4500) = 1800 \cdot 450 - 2(450)^2 = 405{,}000$ sq. meters

(13)




$xy = 25600$ (constraint)

$y = \dfrac{25600}{x}$ (eliminate variable)

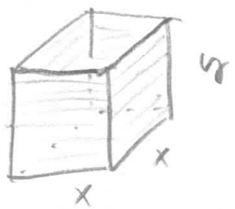
$$\text{Cost} = \underbrace{3\cdot(x+x)}_{\$3/\text{meter for ends}} + \underbrace{1.5y}_{\$1.50/\text{meter for side}}$$

$$= 6x + 1.5\left(\frac{25600}{x}\right) = 6x + \frac{38400}{x} = C(x)$$

$$C'(x) = 6 - \frac{38400}{x^2} = 0, \quad x = \sqrt{\frac{38400}{6}} = \sqrt{6400} = 80 \text{ meters}$$

$$C(80) = \$960$$

(19)




$x^2y = 32 \text{ in}^3$ (constraint)

$y = 32/x^2$ (eliminate variable)

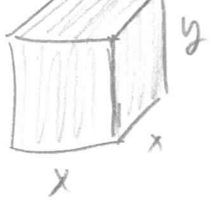
$$\text{Area} = x^2 + 4xy = x^2 + 4x\frac{32}{x^2} = x^2 + \frac{128}{x} = A(x)$$

$$A'(x) = 2x - \frac{128}{x^2} = 0, \quad x = \sqrt[3]{64} = 4 \text{ inches}$$

$$y = \frac{32}{4^2} = 2 \text{ inches}$$

$$\boxed{\underbrace{4\times4}_{\text{base}}\times\underbrace{2}_{\text{height}}}$$

(23)




$x^2 y = 16000\ cm^3$ (constraint)

$y = \dfrac{16000}{x^2}$ (eliminate a variable)

$$\text{Cost} = \underbrace{3(x^2 + x^2)}_{\$3 \text{ per } cm^2 \text{ top/bottom}} + \underbrace{1.5(xy + xy + xy + xy)}_{\$1.50 \text{ per } cm^2 \text{ sides}}$$

$$= 6x^2 + 6xy = 6x^2 + 6x\left(\frac{16000}{x^2}\right)$$

$$= 6x^2 + \frac{96000}{x} = C(x)$$

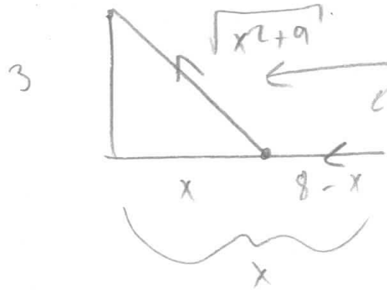
$C'(x) = 12x - \dfrac{96000}{x^2} = 0,\quad x = \sqrt[3]{\dfrac{96000}{12}} = 20\text{ cm}$

$y = \dfrac{16000}{(20)^2} = 40\text{ cm}$

$\underset{\text{base}}{(20 \times 20)} \times \underset{\text{height}}{40}$

$C(20) = 6(20)^2 + \dfrac{96000}{20} = \$7200$

(45)




5 mph along river

2 mph across land

$$\text{time} = \frac{\text{dist}}{\text{rate}}$$

$$T(x) = \frac{8-x}{5} + \frac{\sqrt{x^2+9}}{2}$$

total time    dist    rate    dist    rate

$$T'(x) = -\frac{1}{5} + \frac{x}{2\sqrt{x^2+9}} = 0$$

$$5x = 2\sqrt{x^2+9}$$

$$25x^2 = 4(x^2+9)$$

$$21x^2 = 36$$

$$x = \sqrt{36/21} \approx 1.309 \text{ miles}$$

$$\boxed{8 - x \approx 6.691 \text{ miles along river}}$$

6.3 (15) $\text{Cost} = \underbrace{f\left(\frac{M}{q}\right) + gM}_{\text{manufacturing costs}} + \underbrace{K \cdot q}_{\text{storage cost}} = C(q)$

$\left(\$f/\text{batch}\right) \times \left(\#\text{ of batches}\right)$ $\left(\$K/\text{item}\right) \times \left(\text{max \# of items in storage}\right)$

$$C'(q) = -\frac{fM}{q^2} + K = 0, \quad q = \sqrt{\frac{fM}{K}}$$

(27) $q = 243112 9\, p^{-.06}$

$$E = -\frac{p}{q}\frac{dq}{dp} = \frac{-\cancel{p}}{\cancel{2431129 p^{-.06}}} \cdot (-.06)\cancel{(2431129)}\cancel{p^{-1.06}}$$

$= .06 < 1$ inelastic

(29)

$q = 3751000\, p^{-2.826}$

$$E = -\frac{p}{q}\frac{dq}{dp} = \frac{-p}{3751000\, p^{-2.826}} \cdot (-2.826)(3751000 p^{-3.826})$$

$= 2.826$ elastic