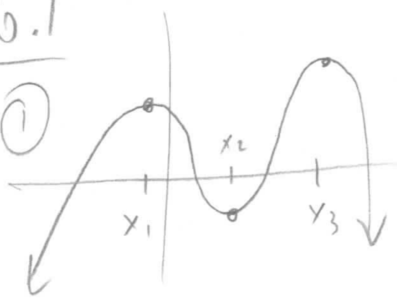
July 24

6.1

1.

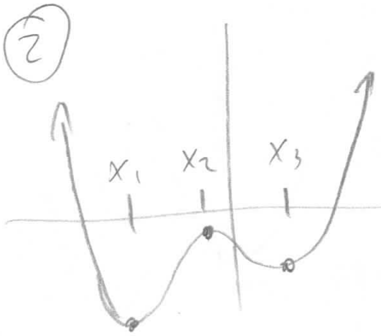


absolute max at x_3

no absolute min

(relative max at x_1, x_3, relative min at x_2)

2.

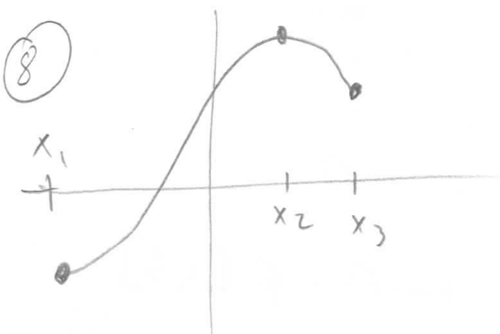


absolute min at x_1

no absolute max

(relative min at x_1, x_3, relative max at x_2)

8.



absolute max at x_2

absolute min at x_1

(relative max at x_2, relative min at x_1, x_3 but not obvious from calculus)

11.

$f(x) = x^3 - 6x^2 + 9x - 8$ on $[0, 5]$

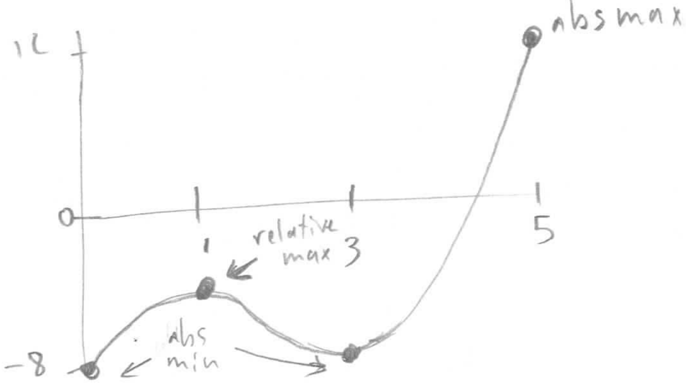
$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-3)(x-1) = 0$ at $x = 1, 3$

crit. number
$$\begin{cases} f(1) = 1 - 6 + 9 - 8 = -4 \\ f(3) = 27 - 54 + 27 - 8 = -8 \end{cases}$$

abs. min of -8 at $x = 0, 3$

end pts.
$$\begin{cases} f(0) = -8 \\ f(5) = 125 - 150 + 45 - 8 = 12 \end{cases}$$

$\leftarrow$ abs. max of 12 at $x = 5$

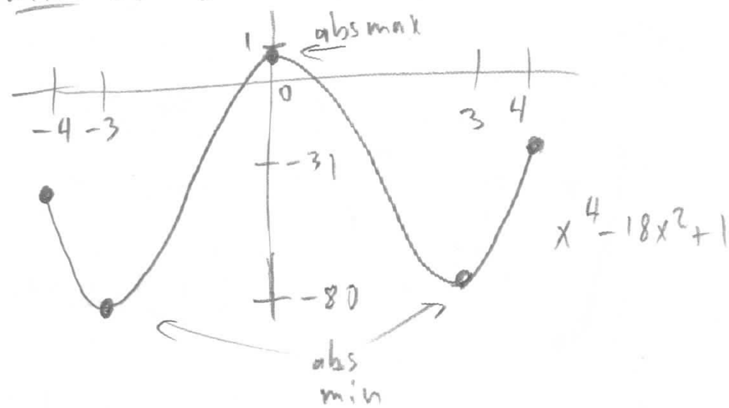


(15) $f(x) = x^4 - 18x^2 + 1$ on $[-4, 4]$

$f'(x) = 4x^3 - 36x = 4x(x^2 - 9) = 4x(x-3)(x+3) = 0$ at $x = -3$, $x = 0$, $x = 3$

crit numbers
$$\begin{cases} f(0) = 1 \leftarrow \text{abs max of } 1 \text{ at } x = 0 \\ f(3) = 81 - 162 + 1 = -80 \leftarrow \text{abs. min of } -80 \text{ at } x = \pm 3 \\ f(-3) = -80 \end{cases}$$

end pts.
$$\begin{cases} f(-4) = 256 - 288 + 1 = -31 \\ f(4) = -31 \end{cases}$$



(19) $f(x) = \dfrac{x-1}{x^2+1}$ on $[1, 5]$

$$f'(x) = \frac{(x^2+1) - (x-1)(2x)}{(x^2+1)^2} = \frac{-x^2 + 2x + 1}{(x^2+1)^2} = 0$$

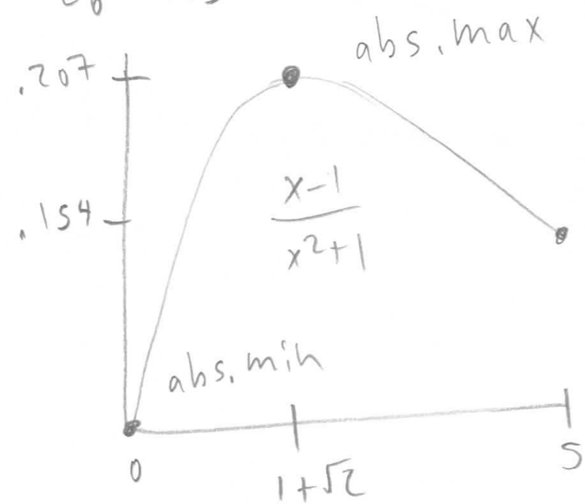
at $x = \dfrac{-2 \pm \sqrt{4+4}}{-2} = 1 \pm \sqrt{2}$

$1 - \sqrt{2} \notin [1, 5]$

$\underline{x = 1 + \sqrt{2} \text{ only crit \#}}$

crit number
$$f(1+\sqrt{2}) = \frac{\sqrt{2}}{(1+\sqrt{2})^2 + 1} = \frac{\sqrt{2}}{1 + 2\sqrt{2} + 2 + 1} = \frac{\sqrt{2}}{4 + 2\sqrt{2}} \approx .207 \leftarrow \text{abs max of } .207 \text{ at } x = 1 + \sqrt{2}$$

end pts
$$\begin{cases} f(1) = 0 \leftarrow \text{abs. min of } 0 \text{ at } x = 1 \\ f(5) = \frac{4}{26} = \frac{2}{13} \approx .154 \end{cases}$$



(25) $f(x) = x^2 - 8\ln x$ on $[1,4]$ $\quad f'(x) = 2x - \frac{8}{x}$

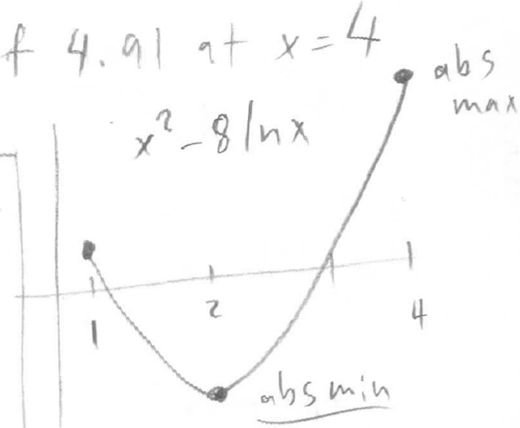
$$2x - \frac{8}{x} = 0$$
$$2x = \frac{8}{x}$$
$$x^2 = 4$$
$$x = \pm 2$$

→ only $x=2$ is in $[1,4]$.

$x=2$ is the only critical number

crit num $\{ f(2) = 4 - 8\ln 2 \approx -1.55$ ← abs min of -1.55 at $x=2$

end pt $\begin{cases} f(1) = 1 \\ f(4) = 16 - 8\ln 4 \approx 4.91 \end{cases}$ ← abs max of 4.91 at $x=4$



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(27) $f(x) = x + e^{-3x}$ on $[-1,3]$

$f'(x) = 1 - 3e^{-3x}$

$f'(x) = 0 \Rightarrow 3e^{-3x} = 1$

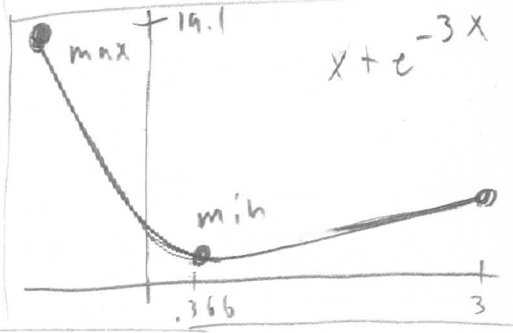
$$e^{-3x} = 1/3$$
$$\ln(e^{-3x}) = \ln\left(\tfrac{1}{3}\right)$$
$$-3x = -\ln 3$$
$$x = \ln 3/3 \approx .366$$

crit $\{ f\left(\frac{\ln 3}{3}\right) = \frac{1+\ln 3}{3} \approx .7$

end pts $\begin{cases} f(-1) \approx 19.1 \\ f(3) \approx 3 \end{cases}$

abs max of 19.1 at $x=-1$

abs min of .7 at $x = \frac{\ln 3}{3}$



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(45) $C(x) = x^3 + 37x + 250$

$\bar{C}(x) = \frac{C(x)}{x} = x^2 + 37 + 250/x$

$\bar{C}'(x) = 2x - 250/x^2 = 0$

$$2x = 250/x^2$$
$$2x^3 = 250$$
$$x = \sqrt[3]{125} = 5$$

(a) $1 \le x \le 10$

$\bar{C}(1) = 258$

$\bar{C}(5) = 112$ ← min

$\bar{C}(10) = 162$

(b) $10 \le x \le 20$

$\bar{C}(10) = 162$ ← min

$\bar{C}(20) = 449.5$

(52) $S(x) = -x^3 + 3x^2 + 360x + 5000 \quad 6 \le x \le 20$

$S'(x) = -3x^2 + 6x + 360 = -3(x^2 - 2x - 120) = -3(x-12)(x+10)$

$x = -10$ not in domain, $x = 12$ only crit. number

$S(6) = 7052$

$S(12) = 8024 \leftarrow$ maximum of 8024 salmon at 12°C

$S(20) = 5400$

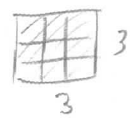
(57) + (58) $A_{sq}(x) = \dfrac{(12-x)^2}{16}$, $A_{circ}(x) = \dfrac{x^2}{4\pi}$, $A_{Tot}(x) = \dfrac{x^2}{4\pi} + \dfrac{(12-x)^2}{16}$, $[0,12]$

$$A'_{Tot}(x) = \frac{x}{2\pi} + \frac{x-12}{8} = \frac{4x + \pi x - 12\pi}{16\pi} = \frac{x(4+\pi) - 12\pi}{16\pi}$$

$$\frac{x(4+\pi) - 12\pi}{16\pi} = 0$$

$$x = \frac{12\pi}{4+\pi} \approx 5.279$$

$A_{Tot}(0) = 9$



$A_{Tot}\left(\dfrac{12\pi}{4+\pi}\right) = 5.041$

$A_{Tot}(12) = 11.46$

absolute max of 11.46

at $x = 12$ (all circle, no square)

absolute min of 5.041

at $x = \dfrac{12\pi}{4+\pi}$ (part square, part circle)