5.3

(7) $f(x) = \dfrac{x^2}{x+1}$ $\quad f'(x) = \dfrac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \dfrac{2x^2+2x-x^2}{(x+1)^2}$

$$= \frac{x^2+2x}{(x+1)^2}$$

$$f''(x) = \frac{(x+1)^2(2x+2) - (x^2+2x)2(x+1)}{(x+1)^4}$$

$$= \frac{2(x+1)^3 - 2x(x+2)(x+1)}{(x+1)^4} = \frac{2(x+1)^2 - 2x(x+2)}{(x+1)^3}$$

$$= \frac{\cancel{2x^2} + \cancel{4x} + 2 - \cancel{2x^2} - \cancel{4x}}{(x+1)^3} = \frac{2}{(x+1)^3}$$

$f(0) = 2$

$f(2) = \frac{2}{27}$

(9)

$f(x) = \sqrt{x^2+4}$ $\quad f'(x) = \frac{1}{2}(x^2+4)^{-1/2}(2x)$

$$= \frac{x}{\sqrt{x^2+4}}$$

$$f''(x) = \frac{\sqrt{x^2+4} - x\,\frac{1}{2}(x^2+4)^{-1/2}(2x)}{x^2+4}$$

$$= \frac{\sqrt{x^2+4} - \dfrac{x^2}{\sqrt{x^2+4}}}{x^2+4} = \frac{x^2+4-x^2}{(x^2+4)^{3/2}}$$

$$= \frac{4}{(x^2+4)^{3/2}}$$

$f(0) = \frac{1}{2\sqrt{2}}$

$f(2) = \frac{1}{4\sqrt{2}}$

(15) $f(x) = \dfrac{\ln x}{4x}$ $\quad f'(x) = \dfrac{4x \cdot \frac{1}{x} - \ln x \cdot 4}{(4x)^2}$

$$= \frac{4(1-\ln x)}{16x^2} = \frac{1-\ln x}{4x^2}$$

$$f''(x) = \frac{4x^2\left(-\frac{1}{x}\right) - (1-\ln x)\,8x}{(4x^2)^2} = \frac{-4x - 8x + 8x\ln x}{16x^4}$$

$$= \frac{-1-2+2\ln x}{4x^3} = \frac{-3+2\ln x}{4x^3}$$

$f(0) =$ undef

$f(2) = \dfrac{-3+2\ln 2}{32}$

(19) $f(x) = 5x^5 - 3x^4 + 2x^3 + 7x^2 + 4$

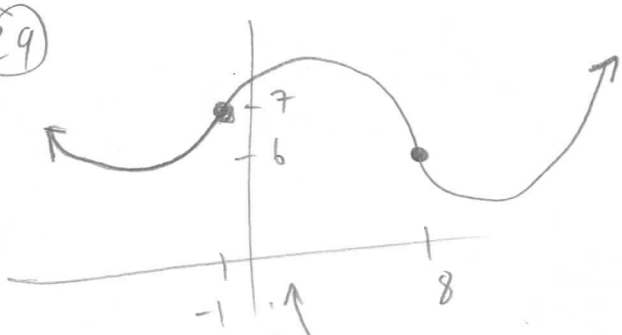
$f'(x) = 25x^4 - 12x^3 + 6x^2 + 14x$

$f''(x) = 100x^3 - 36x^2 + 12x + 14$

$f'''(x) = 300x^2 - 72x + 12$

$f^{(4)}(x) = 600x - 72$ $\quad \left(f^{(5)}(x) = 600,\ f^{(6)}(x) = 0 = f^{(n)}(x),\ n \geq 6 \right)$

(29)

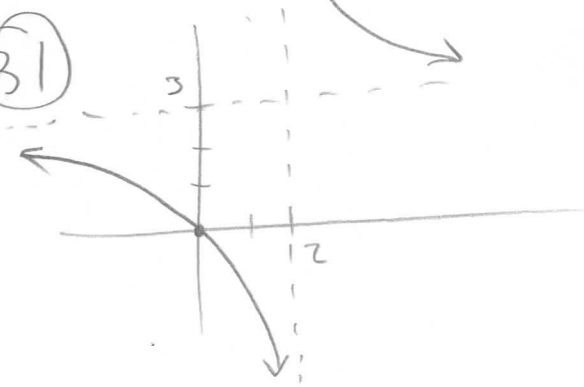


CCU on $(-\infty, -1)$, $(8, \infty)$

CCD on $(-1, 8)$

inflection pts $(-1, 7)$, $(8, 6)$

(31)



CCU on $(2, \infty)$

CCD on $(-\infty, 2)$

no inflection points

(37) $f(x) = \dfrac{3}{x-5}$ $\quad f'(x) = \dfrac{-3}{(x-5)^2}$ $\quad f''(x) = \dfrac{-6}{(x-5)^3}$

$f''(x)$: $+$ | $-$

- f is <u>CCU</u> on $(-\infty, 5)$
- f is <u>CCD</u> on $(5, \infty)$
- <u>no inflection pts</u> (f undefined at $x=5$)

(43) $f(x) = x^{8/3} - 4x^{5/3}$ $\quad f'(x) = \dfrac{8}{3}x^{5/3} - \dfrac{20}{3}x^{2/3}$

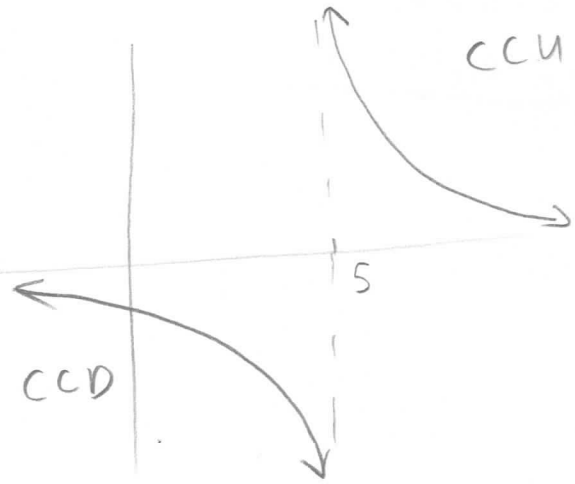
$$f''(x) = \frac{40}{9}x^{2/3} - \frac{40}{9}x^{-1/3} = \frac{40}{9}\left(x^{2/3} - x^{-1/3}\right)$$

- f'' d.n.e. at $x=0$ $\left(\dfrac{1}{\sqrt[3]{x}} = x^{-1/3}\right.$ undefined at $\left.x=0\right)$
- $f''(x)=0$, $\quad x^{2/3} - x^{-1/3} = 0$, $\quad x^{2/3} = x^{-1/3}$, $\quad x = 1$ (multiply both sides by $x^{1/3}$)

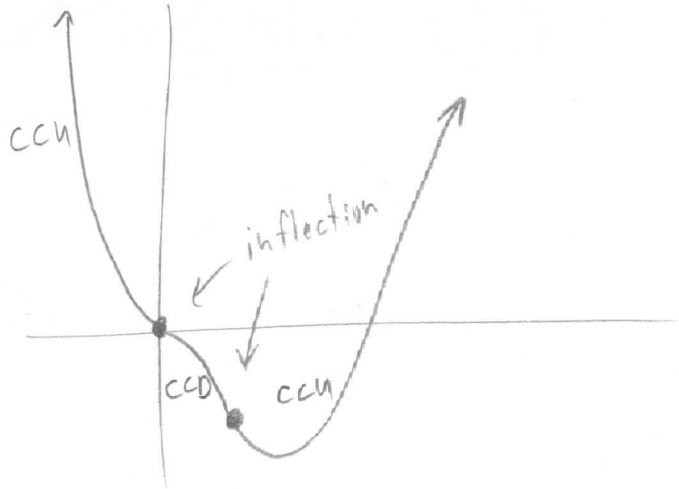
f'': $+$ | $-$ | $+$ (0, 1)

- f is <u>CCU</u> on $(-\infty, 0)$, $(1, \infty)$
- f is <u>CCD</u> on $(0, 1)$
- <u>inflection pts</u> $(0,0)$, $(1,-3)$

graph of (37) $f(x) = \dfrac{3}{x-5}$



graph of (43) $f(x) = x^{8/3} - 4x^{5/3}$



(45) $f(x) = \ln(x^2+1) \quad f'(x) = \dfrac{2x}{x^2+1}$

$$f''(x) = \frac{(x^2+1)2 - 2x(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

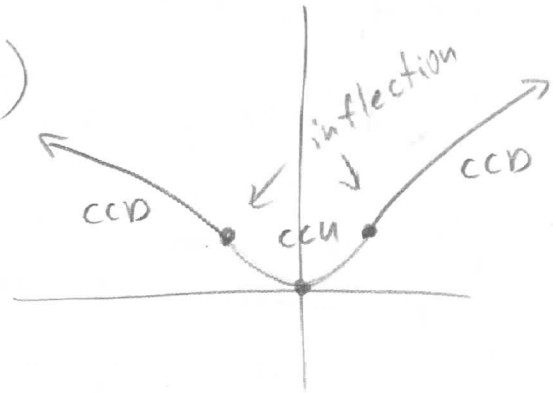
$$= \frac{-2(x^2-1)}{(x^2+1)^2} = \frac{-2(x-1)(x+1)}{(x^2+1)^2}$$

$$f''(x) = \begin{cases} 0 \\ \text{undef } x \end{cases} \quad \text{at } x = 1, -1$$

f'': $-$ | $+$ | $-$; -1, 1

- <u>CCU</u> on $(-1, 1)$
- <u>CCD</u> on $(-\infty, -1), (1, \infty)$

- <u>inflection pts</u> $(-1, \ln 2), (1, \ln 2)$



(75) $R(x) = -.6x^3 + 3.7x^2 + 5x, \quad 0 \le x \le 6$

$R'(x) = -1.8x^2 + 7.4x + 5$

$R''(x) = -3.6x + 7.4 \quad = 0 \text{ at } x = \dfrac{7.4}{3.6} \approx 2.056$

$R(2.056) \approx 20.706 \qquad (2.056, 20.706)$

<u>inflection pt</u>

(84) $G(t) = \dfrac{5200}{1+12e^{-.52t}}$

$$G'(t) = -5200\left(1+12e^{-.52t}\right)^{-2}\left(12\cdot(-.52)e^{-.52t}\right)$$

$$= (5200\cdot 12\cdot .52)\left(1+12e^{-.52t}\right)^{-2}e^{-.52t}$$

$$G''(t) = (5200\cdot 12\cdot .52)\Big[\left(1+12e^{-.52t}\right)^{-2}(-.52)e^{-.52t} + e^{-.52t}\left(-2\left(1+12e^{-.52t}\right)^{-3}\left(12\cdot(-.52)e^{-.52t}\right)\right)\Big]$$

$$= \frac{-(.52)^2\cdot 12\cdot 5200\cdot e^{-.52t}}{\left(1+12e^{-.52t}\right)^3}\left(1-12e^{-.52t}\right)$$

$= 0$ if $1-12e^{-.52t} = 0$

$$1 = 12e^{-.52t}$$

$$\frac{\ln\left(\frac{1}{12}\right)}{-.52} = t = \frac{\ln 12}{.52}$$

$$\approx 4.779$$

$G(4.779) \approx 2600$

After 4.8 yrs, when there are 2600 clams, the grow rate starts decreasing