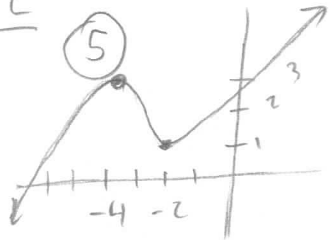
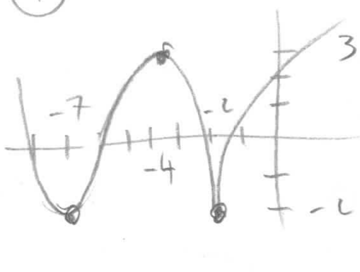
5.2 — July 22

(5)



- local maximum of $3 = f(-4)$ at $x=-4$
- local minimum of $1 = f(-2)$ at $x=-2$

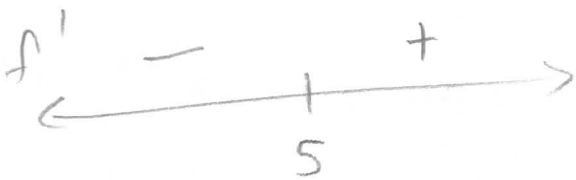
(7)



- local maximum of $f(-4)=3$ at $x=-4$
- local minimum of $f(-2)=-2$ at $x=-2$
- local minimum of $f(-7)=-2$ at $x=-2$

(13) $f(x)=x^2-10x+33$, $f'(x)=2x-10=2(x-5)$

$f'(x)=\begin{cases} 0 \\ \text{undef} \end{cases}$ at $x=5$



f is decreasing on $(-\infty,5)$, increasing on $(5,\infty)$, has a local minimum of $f(5)=8$ at $x=5$

(15) $f(x)=x^3+6x^2+9x-8$, $f'(x)=3x^2+12x+9$

$$= 3(x^2+4x+3)$$
$$= 3(x+3)(x+1)$$

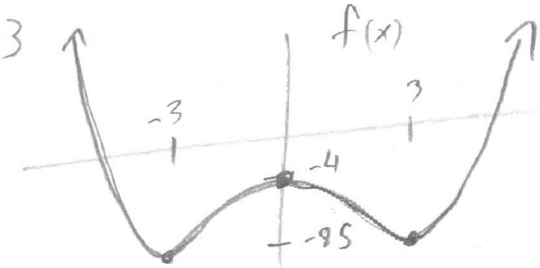
$f'(x)=\begin{cases} 0 \\ \text{undef} \end{cases}$

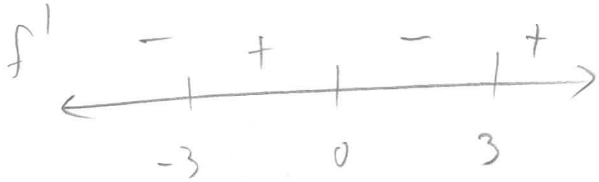
$\Rightarrow x=-3,-1$

- f is increasing $(-\infty,-3)$, decreasing $(-3,-1)$, increasing $(-1,\infty)$.
- f has a local maximum of $f(-3)=-8$ at $x=-3$
- f has a local minimum of $f(-1)=-12$ at $x=-1$

(19) $f(x) = x^4 - 18x^2 - 4$, $f'(x) = 4x^3 - 36x = 4x(x^2-9)$
$= 4x(x-3)(x+3)$

$f'(x) = \begin{cases} 0 \\ \text{undef} \end{cases}$ at $x = 0, 3, -3$





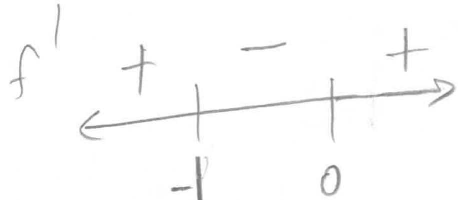
- f has a <u>local minimum</u> of $f(-3) = -85$ at $x = -3$
- f has a <u>local maximum</u> of $f(0) = -4$ at $x = 0$
- f has a <u>local minimum</u> of $f(3) = -85$ at $x = 3$

(23) $f(x) = 2x + 3x^{2/3}$, $f'(x) = 2 + 2x^{-1/3} = 2(1 + x^{-1/3})$

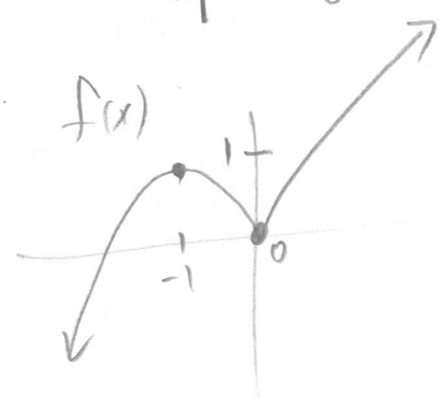
f' is <u>undefined</u> at $x = 0$ $\left(\frac{1}{\sqrt[3]{0}} = 0^{-1/3} \text{ is undefined}\right)$

$f'(x) = 0 \Rightarrow 1 + x^{-1/3} = 0$, $-1 = x^{-1/3}$, $(-1)^3 = (x^{-1/3})^3 = x^{-1} = \frac{1}{x}$

and $x = \frac{1}{(-1)^3} = -1$. <u>critical numbers</u>: $x = 0, -1$



- f has a <u>local maximum</u> of $f(-1) = 1$ at $x = -1$
- f has a <u>local minimum</u> of $f(0) = 0$ at $x = 0$.



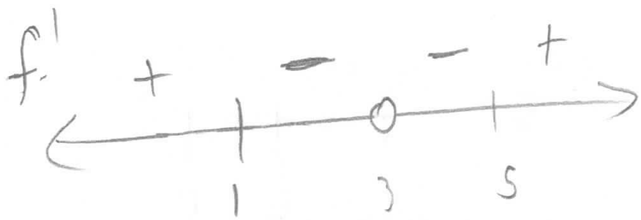
(27) $f(x) = \dfrac{x^2 - 2x + 1}{x - 3}$ (domain $(-\infty, 3) \cup (3, \infty)$)

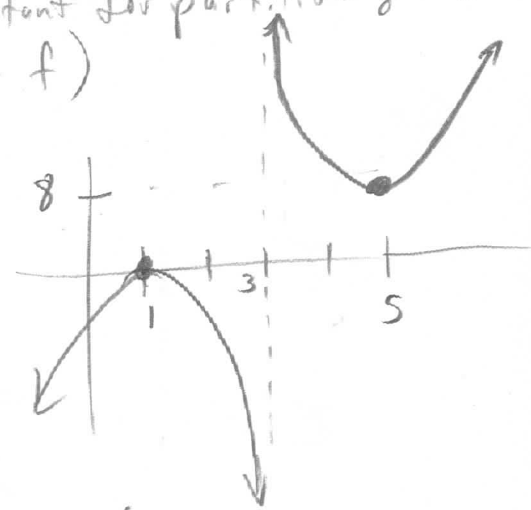
$$f'(x) = \frac{(x-3)(2x-2) - (x^2 - 2x + 1)}{(x-3)^2} = \frac{2x^2 - 8x + 6 - x^2 + 2x - 1}{(x-3)^2}$$

$$= \frac{x^2 - 6x + 5}{(x-3)^2} = \frac{(x-5)(x-1)}{(x-3)^2}$$

$f'(x) = 0$ at $x = 5, 1$

f' is undefined at $x = 3$ ($x = 3$ not in domain of f, so $x = 3$ is not a crit number, but is still important for partitioning the domain of f)

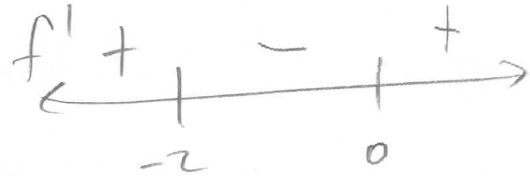




- f has a local maximum of $f(1) = 0$ at $x = 1$
- f has a local minimum of $f(5) = 8$ at $x = 5$

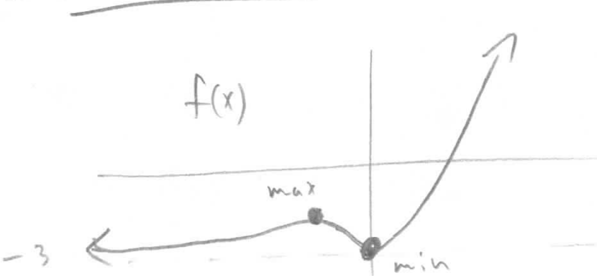
(29) $f(x) = x^2 e^x - 3$, $f'(x) = x^2 e^x + e^x(2x) = xe^x(x+2)$

$f'(x) = \begin{cases} 0 \\ \text{undef} \end{cases}$ at $x = 0, -2$



f has a local maximum of $f(-2) \approx -2.46$ at $x = -2$

f has a local minimum of $f(0) = -3$ at $x = 0$



41. $C(q) = 80 + 18q$, $p = 70 - 2q$

$R(q) = q \cdot p = 70q - 2q^2$

$P(q) = R(q) - C(q) = -2q^2 + 52q - 80$

(a) $P'(q) = -4q + 52 = \begin{cases} 0 \\ \text{undef} \end{cases}$ at $\underline{q = 13}$

(b) $p(13) = 70 - 2(13) = 44$

(c) $P(13) = -2(13)^2 + 52(13) - 80 \approx 258$

49. $C(x) = .002x^3 + 9x + 6912$ (cost)

$\overline{C}(x) = \frac{C(x)}{x} = .002x^2 + 9 + \frac{6912}{x}$ (avg. cost)

$\overline{C}'(x) = .004x - \frac{6912}{x^2}$ (marginal avg. cost)

$\overline{C}'(x) = 0 = .004x - \frac{6912}{x^2} \Rightarrow 6912 = .004x^3$

$x = \left(\frac{6912}{.004}\right)^{1/3}$

$= 120$

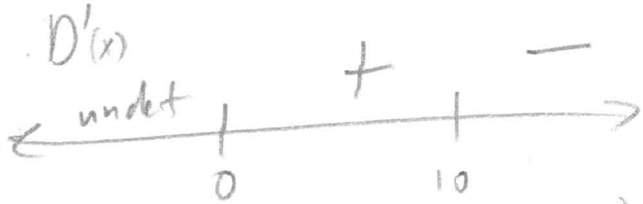
55. $D(x) = -x^4 + 8x^3 + 80x^2$ $(0 \le x)$

$D'(x) = -4x^3 + 24x^2 + 160x$

$= -4x(x^2 - 6x - 40)$

$= -4x(x - 10)(x + 4)$

$= 0$ at $x = -4, 10, 0$



(a score of "10" maximizes attitude change)