4.4

July 18th

(5) $y = -16e^{2x+1} \left(= -16\, f(g(x)),\ f(x) = e^x,\ g(x) = 2x+1\right)$

$y' = -16 \cdot e^{2x+1} \cdot 2 = -32e^{2x+1}$

(15) $y = (x+3)^2 e^{4x} \left(= u(x)\, v(x),\ u(x) = (x+3)^2,\ v(x) = e^{4x}\right)$

$y' = (x+3)^2 (e^{4x})' + e^{4x} ((x+3)^2)'$

$= (x+3)^2 \cdot 4e^{4x} + e^{4x}(2(x+3))$

$= 2(x+3)e^{4x}(2x+7)$

(19) $y = \dfrac{e^x + e^{-x}}{x} \left(= \dfrac{u(x)}{v(x)},\ u(x) = e^x + e^{-x},\ v(x) = x\right)$

$y' = \left[x(e^x + e^{-x})' - (e^x + e^{-x})(x)'\right] / x^2$

$= \left[x(e^x - e^{-x}) - (e^x + e^{-x})\right] / x^2$

$= \left[(x-1)e^x - e^{-x}(x+1)\right] / x^2$

(23)

$f(z) = (2z + e^{-z^2})^2$

$f'(z) = 2(2z + e^{-z^2}) \cdot (2z + e^{-z^2})'$

$= 2(2z + e^{-z^2})(2 + (-2z)e^{-z^2})$

$= 4(2z + e^{-z^2})(1 - ze^{-z^2})$

(27) $y = 3\cdot 4^{x^2+2} \quad \left(= 3\cdot f(g(x)),\ f(x)=4^x,\ g(x)=x^2+2\right)$

$y' = 3(\ln 4)\, 4^{x^2+2}(2x) = 6\cdot \ln 4 \cdot 4^{x^2+2}$

(47) Use logistic function (4.4 example 5), models growth/decay towards an equilibrium.

(a) $G(t) = \dfrac{G_0 m}{G_0 + (m - G_0)e^{-Kmt}}$

$(G_0 = 400,\ m = 5200,\ K = .0001)$

$G(t) = \dfrac{400 \cdot 5200}{400 + 4800 e^{-.52t}} = \dfrac{5200}{1 + 12e^{-.52t}}$

Note: $G'(t) = \left[5200\left(1+12e^{-.52t}\right)^{-1}\right]'$

$= \dfrac{-5200}{(1+12e^{-.52t})^2}\cdot 12(-.52)e^{-.52t}$

$= 32448 e^{-.52t} \big/ (1+12e^{-.52t})^2$

(b) $G(1) \approx 639$ clams

$G'(1) \approx 292$ clams/yr

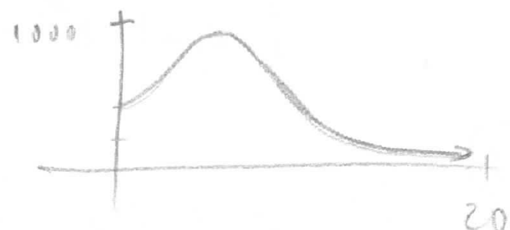
(c) $G(4) \approx 2081$ clams

$G'(4) \approx 649$ clams/yr

(d) $G(10) \approx 4877$ clams

$G'(10) \approx 157$ clams/yr

(e) $G'(t)$



G' increases, then decreases towards 0

4.4 50 $P(t) = .00239\, e^{.0957t}$

(Percentage of people who die at age t)

(a) $P(25) = .026\%$

$P(50) = .286\%$

$P(75) = 3.130\%$

(b) $P'(t) = (.00239)(.0957)\, e^{.0957t}$

$= .00022873\, e^{.0957t}$

$P'(25) = .0025\%/yr$

$P'(50) = .0274\%/yr$

$P'(75) = .300\%/yr$

(c)

4.5

11 $y = -5x \cdot \ln(3x+2)$

$y' = (-5x)\left[\ln(3x+2)\right]' + \ln(3x+2)(-5x)'$

$= -5x \dfrac{1}{3x+2} \cdot 3 + \ln(3x+2) \cdot (-5)$

$= \dfrac{-15x}{3x+2} - 5\ln(3x+2)$

19 $y = \dfrac{3x^2}{\ln x}$, $y' = \dfrac{(\ln x)(3x^2)' - (3x^2)(\ln x)'}{(\ln x)^2}$

$= \dfrac{6x\ln x - 3x^2/x}{(\ln x)^2} = \dfrac{6x}{\ln x} - \dfrac{3x}{(\ln x)^2}$

$= \dfrac{3x}{\ln x}\left(2 - \dfrac{1}{\ln x}\right)$

21 $y = (\ln|x+1|)^4$

$y' = 4(\ln|x+1|)^3 \cdot \dfrac{1}{x+1} \cdot 1$

$= \dfrac{4(\ln|x+1|)^3}{x+1}$

(25) $y = e^{x^2} \ln x$

$y' = (e^{x^2})(\ln x)' + (\ln x)(e^{x^2})'$

$= e^{x^2}/x + (\ln x)\, 2x e^{x^2}$

$= e^{x^2}\left(\frac{1}{x} + 2x \ln x\right)$

(39) $w = \log_8(2^p - 1)$

$w' = \frac{1}{\ln 8\,(2^p - 1)} \ln 2 \cdot 2^p$

$= \frac{\ln 2\; 2^p}{3 \ln 2\,(2^p - 1)} = \frac{2^p}{3(2^p - 1)}$

(41) $f(x) = e^{\sqrt{x}} \ln(\sqrt{x} + 5)$

$f'(x) = (e^{\sqrt{x}})(\ln(\sqrt{x} + 5))' + \ln(\sqrt{x}+5)(e^{\sqrt{x}})'$

$= \frac{e^{\sqrt{x}}}{\sqrt{x}+5}\left(\frac{1}{2}x^{-1/2}\right) + \ln(\sqrt{x}+5)\,\frac{1}{2}x^{-1/2} e^{\sqrt{x}}$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}\left(\frac{1}{\sqrt{x}+5} + \ln(\sqrt{x}+5)\right)$

4.5 43 $f(t) = \dfrac{\ln(t^2+1)+t}{\ln(t^2+1)+1}$

$$f'(t) = \left[\left(\ln(t^2+1)+1\right)\left(\ln(t^2+1)+t\right)' - \left(\ln(t^2+1)+t\right)\left(\ln(t^2+1)+1\right)'\right]$$

$$\cdot \;{}^{1}\!/\!\left(\ln(t^2+1)+1\right)^2$$

$$= \left[\left(\ln(t^2+1)+1\right)\left(\frac{2t}{t^2+1}+1\right) - \left(\ln(t^2+1)+t\right)\left(\frac{2t}{t^2+1}\right)\right] \cdot \frac{1}{\left(\ln(t^2+1)+1\right)^2}$$

$$= \left(\ln(t^2+1)+1+\frac{2t}{t^2+1}-\frac{2t^2}{t^2+1}\right) \cdot \frac{1}{\left(\ln(t^2+1)+1\right)^2}$$

$$= \frac{1}{\ln(t^2+1)+1}\left(1+\frac{2t(1-t)}{(t^2+1)\left(\ln(t^2+1)+1\right)}\right)$$