4.3

July 17th

(7) $f(x) = \frac{x}{8} + 7, \; g(x) = 6x - 1$

$$f(g(x)) = \frac{6x-1}{8} + 7 = \frac{3}{4}x + \frac{55}{8}$$

$$g(f(x)) = 6\left(\frac{x}{8} + 7\right) - 1 = \frac{3x}{4} + 41$$

(Note: $g(f(x)) \neq f(g(x))$ in general!)

(11) $f(x) = \sqrt{x+2}, \; g(x) = 8x^2 - 6$

$$f(g(x)) = \sqrt{(8x^2-6)+2} = \sqrt{8x^2-4} = 2\sqrt{2x^2-1}$$

$$g(f(x)) = 8\left(\sqrt{x+2}\right)^2 - 6 = 8(x+2) - 6 = 8x + 10$$

(13) $f(x) = \sqrt{x+1}, \; g(x) = \frac{-1}{x}$

$$f(g(x)) = \sqrt{-\frac{1}{x} + 1}, \quad g(f(x)) = \frac{-1}{\sqrt{x+1}}$$

$$= \sqrt{\frac{x-1}{x}}$$

(15) $y = (5-x^2)^{3/5} = f(u(x))$ $\begin{cases} f(u) = u^{3/5} \\ u(x) = 5 - x^2 \end{cases}$

(17) $y = -\sqrt{13+7x} = f(u(x))$ $\begin{cases} f(u) = -\sqrt{u} \\ u(x) = 7x + 13 \end{cases}$

(21) $y=(8x^4-5x^2+1)^4 = f(u(x))$ $\begin{cases} f(u)=u^4 \\ u(x)=8x^4-5x^2+1 \end{cases}$

$$y' = 4(8x^4-5x^2+1)^3 \cdot (8x^4-5x^2+1)'$$
$$= 4(8x^4-5x^2+1)^3 \cdot (32x^3-10x) = 8x(16x^2-5)(8x^4-5x^2+1)^3$$

(25) $s(t)=45(3t^3-8)^{3/2} = f(g(t))$ $\begin{cases} f(t)=45t^{3/2} \\ g(t)=3t^3-8 \end{cases}$

$$s'(t) = 45\,\tfrac{3}{2}(g(t))^{1/2}\cdot g'(t)$$
$$= \frac{135}{2}(3t^3-8)^{1/2}(9t^2) = \frac{1215}{2}t^2\sqrt{3t^3-8}$$

(27) $g(t)=-3\sqrt{7t^3-1} = p(q(t))$ $\begin{cases} p(t)=-3\sqrt{t} \\ q(t)=7t^3-1 \end{cases}$

$$g'(t) = -3\,\tfrac{1}{2}(7t^3-1)^{-1/2}\cdot(7t^3-1)'$$
$$= -\frac{3}{2}(7t^3-1)^{-1/2}(21t^2) = \frac{-63t^2}{2\sqrt{7t^3-1}}$$

(39) $y=\dfrac{3x^2-x}{(2x-1)^5}$ $\quad y' \underset{\substack{\uparrow \\ \text{quotient} \\ \text{rule}}}{=} \dfrac{(2x-1)^5[3x^2-x]'-(3x^2-x)[(2x-1)^5]'}{(2x-1)^{10}}$

$[3x^2-x]' = 6x-1$, $\quad [(2x-1)^5]' = 5(2x-1)^4\cdot 2 \leftarrow$ chain rule

$$y' = [(2x-1)^5(6x-1)-(3x^2-x)(10(2x-1)^4)]/(2x-1)^{10}$$
$$= \frac{(2x-1)(6x-1)-10x(3x-1)}{(2x-1)^6} = \frac{-18x^2+2x+1}{(2x-1)^6}$$

(43) (a) $\left.\frac{d}{dx}\right|_{x=1} f(g(x)) = f'(g(1))\cdot g'(1) = f'(2)\cdot\frac{2}{7} = -7\cdot\frac{2}{7} = -2$

(b) $\left.\frac{d}{dx}\right|_{x=2} f(g(x)) = f'(g(2))\cdot g'(2) = f'(3)\cdot\frac{3}{7} = -8\cdot\frac{3}{7} = \frac{-24}{7}$

(45) $f(x) = \sqrt{x^2+16}$ (at $x=3$)

$f'(x) = \frac{1}{2}(x^2+16)^{-1/2}\cdot(2x) = \frac{x}{\sqrt{x^2+16}}$

$f'(3) = \frac{3}{\sqrt{3^2+16}} = \frac{3}{5}$, $f(3) = \sqrt{3^2+16} = 5$

tangent line

$y-5 = \frac{3}{5}(x-3)$, $y = \frac{3}{5}x + \frac{16}{5}$

(49) horizontal tangents at $x=?$

$f(x) = \sqrt{x^3-6x^2+9x+1}$

$f'(x) = \frac{1}{2}(x^3-6x^2+9x+1)^{-1/2}\cdot(3x^2-12x+9)$

$= \frac{3(x^2-4x+3)}{2\sqrt{x^3-6x^2+9x+1}} = 0\,?$

$\Rightarrow x^2-4x+3=0$

$(x-3)(x-1)=0$

$x=3,1$